# Introduction to Digital Electronics

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7/22/13, Midterm 1

EE42/100, Summer 2013

**INSTRUCTIONS**
- You have 100 minutes to complete the exam.
- The exam is closed book, closed notes, closed computer, closed calculator, except one hand-written 8.5" x 11" crib sheet of your own creation.
- Mark your answers ON THE EXAM ITSELF, and CIRCLE or BOX the final results
- Arrange your time wisely, move on if you get stuck on one question, and come back later

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For staff use only.

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1. Phasors (15 points)

Write the following equations in phaser notation

1) \( v_1(t) = 4 \cos(10 \cdot 10^4 t + 10^o) \)

\[
V_1 = 4 \angle 10^o \\
= 4e^{j10^o}
\]

2) \( v_2(t) = 10 \sin(3t - 45^o) \)

\[
V_2 = 10 \angle -45^o \\
= 10e^{-j45^o}
\]

3) \( v_3(t) = 3 \sin(10 \cdot 10^4 t + 180^o) \)

\[
V_3 = 3 \angle 90^o \\
= 3e^{j90^o}
\]

Both \( R \angle \theta \) form and \( re^{j\theta} \) form are right

Each: 5 pts for correct mag and phase

3 pts for only one is correct

1 pt for attempt
2. Nodal Analysis (24 points)

Solving the following circuit using NODAL analysis. Find the value for $V_2$ (unit is V).

Node 1, 3, 5: $V_1 = V_3 = V_5 = 5V$

Node 2 & 4: supernode

$$3I_x + \frac{V_2 - 5}{4} = 7I_x + \frac{V_6 - V_4}{4}$$

$V_4 = V_2 + 6$

Node 6: $\frac{V_6 - V_4}{4} + 5 = 3I_x$

$$I_x = \frac{5 - V_4}{3}$$

$\Rightarrow$ ②, ③, ④ substitute into ①

$$3I_x + V_2 - 5 = 4I_x - 5$$

$$2I_x = V_2 = 5$$

$$5 - (V_2 + 6) = 3V_2$$

$$V_2 = -0.25V$$

+2 going to the right direction trying to reduce eqn. to only $V_2$
3. Equivalent Circuit (12 points)

Consider the unknown, linear circuit in Fig. 1. The box consists of dependent sources, resistors and independent sources. We measure the voltage $V_x$ before the box is attached to anything else and find $V_x = 2V$.

Next, we connect the box to the circuit on Fig. 2 with $R_s = 500 \, \Omega$ and $V_s = 3V$. We measure $i_x = 1mA$:

Now we connect the box to a variable resistor $R_L$,

(a) **What value does $R_s$ need to be in order to receive maximum output power** from the box?
(b) **What is the maximum output power?**

\[ P_{\text{max}} = \frac{(V_{\text{th}})^2}{R_L} \]

\[ = \frac{1}{0.5k\Omega} \]

\[ = 2 \text{ mW} \]

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If said to find "Thevenin equivalence" without any calculation, +1 only get 1pt.

1. $V_{\text{th}} = V_x = 2V \quad +2$

2. $V_{\text{th}} = V_x = 2V \quad +3$

3. (a) $R_{\text{th}} = R_{\text{th}} = 500 \, \Omega \quad +3$

(b) $P_{\text{max}} = \left(\frac{V_{\text{th}}}{2}\right)^2 / R_L$

\[ = \frac{1}{0.5k\Omega} \]

\[ = 2 \text{ mW} \quad +3$

Right approach, wrong number -1
This problem is about design of an integrator circuit, Figure below. Boxes A and B are unknowns for you to put in.

The input is a 5V unit-step source, $V_{in} = 5u(t)$. And the output, $V_{out}$, should integrate $V_{in}$ at a rate of -0.2 V/s and saturates in 25 seconds.

\[
V_{out}(0) = 0
\]

(a) What is $V_{cc}$?

\[
|V_{out}| \leq V_{cc} + 2
\]

\[
V_{out} = -0.2 \frac{V}{s} \times t = -0.2t
\]

$t \leq 25 \text{ sec}$

\[
V_{out} = -0.2 \times 25 = -5 \text{ V}
\]

\[
V_{cc} = 5 \text{ V} + 2
\]

(b) Plot $V_{out}$ in graph below for $t > 0$. Assume all voltages are 0 at $t=0$.

\[
V_{out}(t) = -0.2t
\]

saturates at $-5 \text{ V}$
(c) What is the RC constant needed for boxes A and B?

Integrator

\[ V_{out} = -\frac{1}{RC} \int V_{in} \, dt \]

\[ = -\frac{1}{RC} \int 5 \, dt \]

\[ = -\frac{5}{RC} \]

\[ RC = \frac{5}{0.2} = 25 \]

+1 for attemption
+1 for correct eqn. Integration
+2 for correct answer.

(d) You have an unlimited supply of 10MΩ resistors and 1μF capacitors. How will you use them to get the values needed for boxes A and B?

\[ R = 5 \, M\Omega, \quad C = 5 \, \mu F \]

+2 for correct R and C to give RC = 25

+3 for resistors
series/parallel relation

+3 for capacitors
series/parallel relation

2 if didn't use 10MΩ/1μF
used other combinations.

Only get 4 pts if RC = 25
and used R.C * at the right position, but single R.C
5. RLC Circuits (20 points)

(a) A RLC series circuit is connected to a unit step input $u(t)$. Use KVL and write a second order equation of $V_c(t)$ in response to the input.

\[ i = i_c = C \frac{dv_c}{dt} + \alpha \text{ for current} \]

\[ V_R + V_L + V_c = u(t) + \frac{d}{dt} \text{KCL} \]

\[ i_c \times L + L \frac{dv_c}{dt} + V_c = u(t) \]

\[ C \frac{dv_c}{dt} + L \frac{dv_c}{dt} + V_c = u(t) + 2 \text{ final form} \]

\[ \frac{dv_c}{dt} + \frac{dv_c}{dt} + V_c = 1 \text{ okay if } u(t) = 1 \]

(b) What is $\alpha$, $\omega_0$?

\[ \alpha = \frac{R}{2L} = \frac{1}{2 \times 1} = 0.5 \text{ Np/sec} + 2 \]

\[ \omega_0 = \sqrt{\frac{1}{LC}} = 1 \text{ rod/sec} + 2 \text{ unit missing - 0.5} \]
(C) Is this overdamped, underdamped, or critically damped?

4 pt

\[ \alpha < \omega_0 \]

underdamped

+2 if correct answer

(d) What is the value of \( V_c(\infty) \) as time goes to \( \infty \)?

4 pt

open circuit: +2 for right direction

\[ V_c(\infty) = V(0+) = V + 2 \text{ for correct answer} \]
6. Resistor Networks (5 points)

A regular octahedron is a Platonic solid composed of eight equilateral triangles, four of which meet at each vertex.
Consider a resistor network connected in the form of octahedron with identical resistors of resistance R. What is the equivalent resistance between points A and B, which are opposite corners of an octahedron?
Hint: Think about Wheatstone bridge. It should NOT include complex calculations.