Problem 1 (15 points)
For all of the problems below,

\[ V_1 = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases} \]

a) Consider the circuit below. (5 points)

Provide a symbolic expression for the following at \( t = \infty \):

<table>
<thead>
<tr>
<th>Solution:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{C1} ) = ( \frac{R_2}{R_1+R_2} ) [V]</td>
<td>( i_{C1} = 0 )</td>
</tr>
<tr>
<td>( V_{L1} ) = 0</td>
<td>( i_{L1} = \frac{1}{R_1+R_2} ) [A]</td>
</tr>
<tr>
<td>( V_{C2} ) = ( \frac{R_2}{R_1+R_2} ) [V]</td>
<td>( i_{C2} = 0 )</td>
</tr>
<tr>
<td>( V_{L2} ) = 0</td>
<td>( i_{L2} = 0 )</td>
</tr>
<tr>
<td>( V_{C3} ) = ( \frac{R_2}{R_1+R_2} ) [V]</td>
<td>( i_{C3} = 0 )</td>
</tr>
</tbody>
</table>

at \( t=\infty \), \( C \) open, \( L \) short

\[ V = V_1 \]
b) Consider the circuit below. (5 points)

Provide a symbolic expression for the following at \( t = \infty \):

<table>
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<tbody>
<tr>
<td>( V_{C1} = \frac{R_2}{R_1+R_2} ) [V]</td>
<td>( i_{C1} = 0 )</td>
</tr>
<tr>
<td>( V_{L1} = 0 )</td>
<td>( i_{L1} = \frac{1}{R_1+R_2} ) [A]</td>
</tr>
<tr>
<td>( V_{C2} = \frac{R_3}{R_1+R_2} ) [V]</td>
<td>( i_{C2} = 0 )</td>
</tr>
<tr>
<td>( V_{L2} = 0 )</td>
<td>( i_{L2} = 0 )</td>
</tr>
<tr>
<td>( V_{C3} = 0 ) OR ( \frac{R_2}{R_1+R_2} \cdot \frac{C_4}{C_3+C_4} ) [V]</td>
<td>( i_{C3} = 0 )</td>
</tr>
</tbody>
</table>
c) Since real capacitors have a leakage resistance, let's add that in. Consider the circuit below. (5 points)

\[ V_1 \]

\[ R_1 \]
\[ L_1 \]
\[ C_1 \]
\[ R_A \]
\[ R_B \]
\[ C_2 \]
\[ R_C \]
\[ R_D \]

Provide a symbolic expression for the following at \( t = \infty \):

<table>
<thead>
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<th>Solution:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{C1} = \frac{R_{th_1}}{R_{th_1} + R_1} ) [ V ]</td>
</tr>
<tr>
<td>( V_{L1} = 0 )</td>
</tr>
<tr>
<td>( V_{C2} = \frac{R_{th_1}}{R_{th_1} + R_1} ) [ V ]</td>
</tr>
<tr>
<td>( V_{L2} = 0 )</td>
</tr>
<tr>
<td>( V_{C3} = \frac{R_{th_1}}{R_{th_1} + R_1} \cdot \frac{R_C}{R_C + R_D} ) [ V ]</td>
</tr>
</tbody>
</table>

Extra Space

where

\( R_{th_1} = R_A \parallel R_2 \parallel R_B \parallel (R_C + R_D) \)

\( R_{th_2} = R_2 \parallel R_B \parallel (R_C + R_D) \)
There is a theory which states that if ever anyone discovers exactly what the Universe is for and why it is here, it will instantly disappear and be replaced by something even more bizarre and inexplicable. There is another theory which states that this has already happened. — Douglas Adams, *The Restaurant at the End of the Universe*

**Problem 2 (30 points)**
In each section below, you will find a circuit where several components have been left “blank” for you to fill in.
- The op amp is ideal in each case.
- In all cases, the opamp is connected to ideal voltage sources of value $V_1$ and $V_2$, where shown.
- Voltages are in [V] and currents are in [A]; constants are in the appropriate units.
- There should be no need for positive feedback in your solutions.

**IMPORTANT:** Partial credit will only be given if you show your work and solution (we can’t give you partial points for blank circuits!).

You have at your disposal as many of the following components as you like:

<table>
<thead>
<tr>
<th>Component</th>
<th>Illustration</th>
</tr>
</thead>
<tbody>
<tr>
<td>an open circuit</td>
<td><img src="image1.png" alt="Open Circuit" /></td>
</tr>
<tr>
<td>a short circuit</td>
<td><img src="image2.png" alt="Short Circuit" /></td>
</tr>
<tr>
<td>a resistor (you choose the value)</td>
<td><img src="image3.png" alt="Resistor" /></td>
</tr>
<tr>
<td>a capacitor</td>
<td><img src="image4.png" alt="Capacitor" /></td>
</tr>
<tr>
<td>an inductor</td>
<td><img src="image5.png" alt="Inductor" /></td>
</tr>
<tr>
<td>a DC voltage source (you choose a fixed value)</td>
<td><img src="image6.png" alt="DC Source" /></td>
</tr>
<tr>
<td>a load (whose value is unknown but high enough that the amp can drive it)</td>
<td><img src="image7.png" alt="Load" /></td>
</tr>
</tbody>
</table>
a) Add components in every box so that $|V_{out}/V_1| = 1$ and the voltage $V_2$ plays no role. (7.5 points)

b) Add components every box so that $V_{out} = 5V_1 - 10V_2$. (7.5 points)

$b_0 = (1 + \frac{R_2}{R_1}) (\frac{R_4}{R_3 + R_4}) U_1 - (\frac{R_2}{R_1}) V_2 = 5V_1 - 10V_2$

$\frac{R_2}{R_1} = 10$
c) Add components every box so that \( V_{\text{out}} = -5 \frac{dv_2}{dt} \) and the voltage \( V_1 \) plays no role. (7.5 points)

\[ U_0 = -RC \frac{dV_{\text{in}}}{dt} \]
\[ RC = 5 \]

\[ \frac{L}{R} = 5 \]

d) Add components every box so that \( i_{fn} = 10V_2 \) and the voltage \( V_1 \) plays no role. (7.5 points)

\[ \frac{1}{R} = 10 \Rightarrow R = 0.15 \Omega \]
“Minor magicians take pains to fit this traditional wizardly bill. By contrast, the really powerful magicians take pleasure in looking like accountants.”

— Jonathan Stroud, *The Amulet of Samarkand*

**Problem 3 (25 points)**

Consider the circuit below. The switch is closed until $t = 20$ s, then opened.

**Solution:**

<table>
<thead>
<tr>
<th><strong>a) Provide a symbolic expression for $i_1$ at $t &lt; 20$ s? (2.5 points)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_1 = \frac{V_S}{R_1} - \frac{1}{R_1} \left( \frac{V_S}{R_1} + \frac{V_0}{R_3} \right) \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$, $t &lt; 20$ s</td>
</tr>
</tbody>
</table>

$L$ is a short. $\frac{V_S - V_x}{R_1} = \frac{V_x}{R_2} + \frac{V_x - V_0}{R_3}$

$V_x = \left( \frac{V_S}{R_1} + \frac{V_0}{R_3} \right) \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$

**b) Provide a symbolic expression for $i_2$ at $t < 20$ s? (2.5 points)**

<table>
<thead>
<tr>
<th><strong>Solution:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_2 = \frac{1}{R_2} \left( \frac{V_S}{R_1} + \frac{V_0}{R_3} \right) \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$, $t &lt; 20$ s</td>
</tr>
</tbody>
</table>

$i(t) = i_1 - i_2 = \frac{V_S}{R_1} + \left( -\frac{1}{R_2} - \frac{1}{R_1} \right) \left( \frac{V_S}{R_1} + \frac{V_0}{R_3} \right) \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$
c) Provide a symbolic expression for $i_1$ at $t \geq 20$ s? (10 points)

**Solution:**

\[ i_1 = 0 \text{ A} \]

\[ t \geq 20 \text{ s}, \quad i(t) = i(\infty) + \left[ i(\infty) - i(\infty) \right] e^{-\frac{(t-20)}{\tau}} \]

\[ \tau = \frac{L}{R_2 + R_3} \]

\[ i_2 = -i(t) \]

d) Provide a symbolic expression for $i_2$ at $t \geq 20$ s? (10 points)

**Solution:**

\[ i_2 = -i(\infty) + \left[ i(\infty) - i(\infty) \right] e^{-\frac{(t-20)}{\tau}}, \quad t \geq 20 \text{ s} \]

\[ i(\infty) = -\frac{V_0}{R_2 + R_3} \]

\[ i(20) = \frac{V_0}{R_1} - \left( \frac{1}{R_1 + R_2} \right) \left( \frac{V_0}{R_2} + \frac{V_0}{R_3} \right) \frac{1}{R_1 + R_2 + R_3} \]

\[ \tau = \frac{L}{R_2 + R_3} \]
"And yet, will we ever come to an end of discussion and talk if we think we must always reply to replies? For replies come from those who either cannot understand what is said to them, or are so stubborn and contentious that they refuse to give in even if they do understand."

— St. Augustine of Hippo, *City of God*

**Problem 4 (30 points)**

For all the circuits below,

\[ V_1(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases} \]

Provide an expression for \( v_{out}(t) \) for \( t \geq 0 \).

a) (5 points)

![Circuit Diagram A](image1.png)

\[ v_{out}(t) = 1 - e^{-t/(RC)} \]

b) (5 points)

![Circuit Diagram B](image2.png)

\[ v_{out}(t) = e^{-t/T} = e^{-t/(RC)} \]
c) (5 points)

\[ v_{out}(t) = 1V \]

\[ \text{Note: } \text{This is only true when } t = 8(t), C \text{ [Dirac delta function times } C] \]

- \( 8(t) \) is an infinitely high, infinitely thin spike at \( t=0 \) such that \( \int_{-\infty}^{\infty} 8(t) dt = 1 \)
- \( V = \frac{1}{C} \int_{-\infty}^{t} \phi(t) dt = \frac{1}{C} \int_{-\infty}^{t} 8(t) C dt = 1V \)
- No such \( 8(t) \) in real life.

\[ \begin{align*}
  t<0 & \quad \Rightarrow \quad V_c(0)=0 \\
  t>0 & \quad \Rightarrow \quad V_c(\infty)=0 \quad \text{and} \quad V_c(t)=0 \\
  \text{Vout} & = \text{Vin} - V_c \\
 & = 1V
\end{align*} \]

d) (5 points)

\[ v_{out}(t) = 1V \]
e) (5 points)

\[ v_{\text{out}}(t) = 1 - e^{-tR/L} \quad [\text{V}] \]

f) (5 points)

\[ v_{\text{out}}(t) = e^{-tR/L} \quad [\text{V}] \]

\[ t < 0, \quad i(0) = 0 \]
\[ t > 0 \]
\[ i(t) = \frac{1}{L} \int_0^t v(t') dt' - \frac{R}{L} e^{-t/t}, \quad t = \frac{L}{R} \]
\[ v_{\text{out}}(t) = i(t) \cdot R = 1 - e^{-t/L} \]

\[ t < 0, \quad i(0) = 0 \]
\[ t > 0 \]
\[ i(t) = \frac{1}{L} \int_0^t v(t') dt' - \frac{R}{L} e^{-t/t}, \quad t > 0 \]
\[ t = \frac{L}{R} \]
\[ v_{\text{out}}(t) = \frac{L \frac{d}{dt} i(t)}{R} \]
\[ = L \left( \frac{1}{L} \frac{1}{R} e^{-t/L} \right) \]
\[ = e^{-t/L} \]