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University of California at Berkeley  
Electrical Engineering and Computer Science  
EE105 Midterm Examination #2  
April 9, 2015  
(80 minutes)

CLOSED BOOK; Two standard 8.5” x 11” sheets of notes (both sides) permitted

IMPORTANT NOTES

- Read each problem completely and thoroughly before beginning to work on it
- Summarize all your answers in the boxes provided on these exam sheets
- Show your work in the space provided so we can check your work and scan for partial credit
- Remember to put your name in the space above

<table>
<thead>
<tr>
<th>Problem #</th>
<th>Points Possible</th>
<th>Score</th>
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<tbody>
<tr>
<td>1</td>
<td>14</td>
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<tr>
<td>2</td>
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<td>3</td>
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<td>4</td>
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<td>16</td>
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<tr>
<td>Total</td>
<td>72</td>
<td>72</td>
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</table>
1. **Circuit Analysis (14 points)** Find the current in each element. Show each step in your solution to receive full credit. Two copies of the same schematic are shown in case you need more than one.

Extraordinary node is one with at least 3 branches. Thus, there are three as indicated. Choose one as a ground node with the others unknown known voltages as shown. Then, account for the two voltage sources as shown.

\[ \text{KCL @ } A: \quad 24 + 19V_A - 16V_B = 0 \quad \cdots (1) \]

\[ \text{KCL @ } B: \quad -54 - 8V_A + 11V_B = 0 \quad \cdots (2) \]

Solving gives:

\[
\begin{align*}
V_A &= 7.41 \text{V} \\
V_B &= 10.30 \text{V}
\end{align*}
\]

\[
\begin{align*}
I_{R1} &= \frac{(V_A + 4) - V_B}{2} = 0.55 \text{ A} \\
I_{R2} &= 10 - V_B = -0.08 \text{ A} \\
I_{R3} &= \frac{V_A - V_B}{6} = -0.48 \text{ A} \\
I_{R4} &= \frac{V_A - 0}{8} = 0.93 \text{ A} \\
I_{VA} &= I_{R1} = 0.55 \text{ A} \\
I_{VB} &= I_{R2} = -0.08 \text{ A}
\end{align*}
\]
2. Bipolar Junction Transistor DC Bias (18 points). Find expressions for the Quiescent Points for Q1-Q3 in terms of $\beta$ and the reference current $I$. All transistors are identical and operate in the Forward Active Region (F.A.R.). Neglect the Early effect.

\[
F.A.R. \Rightarrow I_C = I_S (e^{V_{BE}/V_T} - 1)
\]

\[
V_{BE1} = V_{BE2} \quad I_C1 = I_C2
\]

KCL @ A:
\[
I = I_C1 + I_E3 - - - - - - - - - - 1
\]

KCL @ B:
\[
I_E3 = I_B1 + I_B2 + I_C2
\]

\[
= \frac{I_C1}{\beta} + \frac{I_C1}{\beta} + I_C1 = I_C1 \left(1 + \frac{2}{\beta}\right) \quad 2
\]

Now, substitute 2 into 1:
\[
I = I_C1 + I_E3 = I_C1 + \frac{I_E3}{\beta+1} = \left(\frac{\beta+2+2/\beta}{\beta+1}\right)I_C1
\]

\[
\therefore \quad I_C1 = I_C2 = \frac{(\beta+1)I}{(\beta+2+2/\beta)} \quad I_C3 = \frac{\beta}{\beta+1}I_E3
\]

\[
V_{CE1} = V_C1 - V_E1 = (V_{BE2}+V_{BE3}) - 0 = 2V_{BE}
\]

\[
V_{CE2} = V_C2 - V_B2 = V_{BE2} - 0 = V_{BE}
\]

\[
V_{CE3} = V_C3 - V_E3 = V_C2 - V_{BE}
\]

<table>
<thead>
<tr>
<th>Transistor</th>
<th>Equations</th>
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<tbody>
<tr>
<td>For Q1:</td>
<td>$I_{C1} = \left(\frac{\beta+1}{\beta+2+2/\beta}\right)I$</td>
</tr>
<tr>
<td></td>
<td>$V_{CE1} = 2V_{BE}$</td>
</tr>
<tr>
<td>For Q2:</td>
<td>$I_{C2} = \left(\frac{\beta+1}{\beta+2+2/\beta}\right)I$</td>
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<tr>
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<td>$V_{CE2} = V_{BE}$</td>
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<tr>
<td>For Q3:</td>
<td>$I_{C3} = \left(\frac{\beta+2}{\beta+2+2/\beta}\right)I$</td>
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<tr>
<td></td>
<td>$V_{CE3} = V_C2 - V_{BE}$</td>
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</tbody>
</table>
3. **Dominant Pole Approximation:** [24 points]. Derive an expression for the dominant pole frequency, \( \omega_p \), of the circuit below using the open-circuit time constant (OCTC) method. Assume \( M_1 \) and \( M_2 \) operate in the saturation region. Do not use the Miller Effect. (Note: Next page is blank and available for your solution if you need more space.)

First, draw the complete small-signal model for \( M_1 \) and \( M_2 \).

For \( M_2 \):
- \( C_{gs2} \) is shorted — neglect
- \( g_{m2} \) is zero because \( V_{sg2} = 0 \) — neglect
- \( C_{gd2} \) from \( V_{out} \) to \( \text{gnd} \); \( R_{02} \) from \( V_{out} \) to \( \text{gnd} \)

Redraw simplified circuit:

\[
\omega_p = \left[ R C_{gs1} + R_0 C_0 + (R_0 + R + g_{m1} R_0 R) C_{gd2} \right]^{-1}
\]

Where \( R_0 = R_{01} || R_{02} \)

And \( C_0 = C_L + C_{gd2} \)
3. (cont – Blank page for solution)  * Now use OCTC method *

(i) Find driving point resistance for $C_{gs1}$ with $C_{gd1}$ and $C_{gd2}$ open circuited:

$$\mathbf{V_{test}} = R \quad \Rightarrow \mathbf{Z_1 = R \cdot C_{gs1}}$$

(ii) Driving point resistance for $C_{o}$ with $C_{gs1}$ and $C_{gd}$ open:

$$\mathbf{Z_2 = R_o \cdot C_o}$$

(iii) Driving point resistance for $C_{gd1}$ with $C_{gs1}$ and $C_0$ open:

$$\mathbf{Z_3 = C_{gd1}} \left( R_o + R + \frac{g_m \cdot R \cdot R_o}{R_o} \right)$$

- KCL at $V_{out}$: $i_{test} + g_m \cdot R \cdot i_{test} + g_o \cdot i_{test} + R \cdot i_{test} - g_o \cdot V_{test} = 0$

$$\mathbf{\frac{V_{test}}{i_{test}} = R_o (1 + g_m \cdot R + g_o \cdot R)}$$

$$\mathbf{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$
4. Dominant Pole Approximation: [16 points]. Derive an expression for the dominant pole frequency, \( \omega_p \), of the circuit below using the Miller Effect, as appropriate. Assume \( M_1 \) and \( M_2 \) operate in the saturation region. (Note: Next page is blank and available for your solution if you need more space.)

Recall Miller Effect:

\[
\frac{C}{1-a} \frac{1}{a} \Rightarrow C \text{ for } a \text{ large.}
\]

Here is the simplified circuit from problem 3:

\[
\begin{align*}
V_{in} & \rightarrow R \rightarrow (A) \rightarrow (B) \rightarrow V_{out} \\
& \quad \frac{C_{gs1}}{V_{gs1}} \quad \frac{9m_1}{V_{gs1}} \quad \frac{R_o}{R_0} \quad \frac{C_0}{V_{in}}
\end{align*}
\]

Note: \( C_{gd1} \) is connected between (A) and (B) so we need to find the voltage gain from (A) to (B):

\[
\begin{align*}
V_{in} & \rightarrow R \rightarrow (A) \rightarrow (B) \rightarrow V_{out} \\
& \quad \frac{-V_{gs1}}{9m_1} \quad \frac{R_o}{R_0} \quad \frac{C_0}{V_{in}}
\end{align*}
\]

Expression (no limits):

\[
\omega_p = \left[ R (C_{gs1} + C_{gd1}(1 + 9m_1 R_o)) + R_o (C_0 + C_{gd1}) \right]^{-1}
\]

Factor to be like page 4 solution:

\[
\omega_p = \left[ RC_{gs1} + R_o C_0 + (R_o + R + 9m_1 RR_o) \right]^{-1}
\]

SAME RESULT!
4. (cont – Blank page for solution)

\[ R \]

\[ V_{in} \]

\[ C_{gs1} \quad \frac{+}{U_{gs1}} \quad C_{gd1} (1 - A_t) \]

\[ Q \]

\[ 9m \quad U_{gs1} \]

\[ R_0 \]

\[ \frac{+}{C_0} \]

\[ C_{gd1} (1 - A_t) \]

\[ C_{x} \]

\[ C_{y} \]

\[ U_{out} \]

\( (e') \) Driving point resistance for \( C_x \):

\[ R \]

\[ + \]

\[ U_{test} \]

\[ - \]

\[ V_{test} \]

\[ R \]

\[ U_{test} = R \quad \therefore \quad Z_4 = R C_x = R \left( C_{gs1} + C_{gd1} (1 + 9m R_0) \right) \]

\( (e') \) Driving point resistance for \( C_y \):

\[ R \]

\[ + \]

\[ U_{gs1} \]

\[ - \]

\[ Q \]

\[ 9m \quad U_{gs1} \]

\[ R_0 \]

\[ + \]

\[ V_{test} \]

\[ V_{gs1} = 0 \quad \therefore \quad 9m \cdot U_{gs1} = 0 \]

\[ \therefore \quad V_{test} = R_0 \quad \therefore \quad Z_5 = R_0 \left( C_0 + C_{gd1} \right) \]

\[ W_P = \frac{1}{Z_4 + Z_5} \]