Midterm \#2 Exam- Physics 7B-002 - Tue April $7^{\text {th }} 2015$ (7-9pm) Spring 2015 - UC Berkeley - Eric Corsini

Carefully read the following
On the cover of your green/blue book write (legibly) in that order

- First and Last Name

Name:
SID
Discussion Section
GSI.

- SID number
- Feb 24,2015
- Physics 7B - MT1
- Lec 2
- [your discussion section \#]
- GSI name
- Row and seat number

There are seven (5) problems

- All problems are worth the same number of points each
- Point values are assigned to some of the problem subparts


## Strategy

- Start by reading all problems carefully
- Attempt all problems, show your work, box your answers, check units.


## How to maximize partial credit and avoid losing points

- You are writing to a jury of five graders; your work must be clear to them not just to you
- Carefully show your reasoning so that the grader can be sure you derived the answer as opposed to guessing or relying on a solved problem or on memory of a solved problem.
- Show the logical steps of your work and reasoning, and write legibly; this will enable the grader to give partial credit.
- No credit will be given for unjustified answers even if it is the correct answer
- Cross out any part of the solution you do not want the grader to grade.
- Give your answers in terms of the known variables given in each question
- It may be that the answer does not depend on ALL the given variables in that question.
- All required diagrams must be drawn in the blue book and be at least $1 / 2$ page of the blue book. Be accurate in your drawing; for example, lines or vectors meant to be parallel should be drawn parallel; similarly if they are meant to be perpendicular; it does not have to be perfect but your drawing should clearly indicate what you mean.
- Clearly indicate your choice and orientation of axes; however, when given, use the same axes orientation as shown in the problem-figure.
- If you are stuck on a problem write down how you would proceed if you had more time.
- If you believe the answer you derived is incorrect, say why.
- Check the units of your final answers
- Box your final answer in each of the question in each of the given problem.
- Box both numerical and algebraic answers when both are required.


## Exam Policies

- Raise your hand if you have any question - however, an answer will not necessarily be provided.
- Anyone who uses a wireless-capable device will receive zero on the exam.
- Cell phones must be turned OFF and placed in your backpack (not in your pocket!).
- No calculators allowed
- The only items allowed on your desk are: pencils, eraser, blue book, student picture ID.
- You must show your student ID upon request
- Do not reach inside your backpack during the exam (it will be deemed an attempt at cheating and will result in a zero score for that exam; a report will also be made to the university)


## Keep the test face up until instructed to start

$$
\begin{aligned}
& \vec{F}=\frac{Q_{1} Q_{2}}{4 \pi \epsilon_{0} r^{2}} \hat{r} \\
& \vec{F}=Q \vec{E} \\
& \vec{E}=\int \frac{d Q}{4 \pi \epsilon_{0} r^{2}} \hat{r} \\
& \rho=\frac{d Q}{d V} \\
& \sigma=\frac{d Q}{d A} \\
& \lambda=\frac{d Q}{d l} \\
& \vec{p}=Q \vec{d} \\
& \vec{\tau}=\vec{p} \times \vec{E} \\
& U=-\vec{p} \cdot \vec{E} \\
& \Phi_{E}=\int \vec{E} \cdot d \vec{A} \\
& \oint \vec{E} \cdot d \vec{A}=\frac{Q_{\text {encl }}}{\epsilon_{0}} \\
& \Delta U=Q \Delta V \\
& V=-\int \vec{E} \cdot d \vec{l} \\
& V=\int \frac{d Q}{4 \pi \epsilon_{0} r} \\
& \vec{E}=-\vec{\nabla} V \\
& Q=C V \\
& C_{e q}=C_{1}+C_{2}(\text { In parallel }) \\
& \frac{1}{C_{e q}}=\frac{1}{C_{1}}+\frac{1}{C_{2}}(\text { In series }) \\
& \epsilon=\kappa \epsilon_{0} \\
& U=\frac{Q^{2}}{2 C} \\
& U=\int \frac{\epsilon_{0}}{2}|\vec{E}|^{2} d V \\
& I=\frac{d Q}{d t} \\
& \Delta V=I R \\
& R=\rho \frac{l}{A} \\
& \rho(T)=\rho\left(T_{0}\right)\left(1+\alpha\left(T-T_{0}\right)\right) \\
& P=I V \\
& I=\int \vec{j} \cdot d \vec{A} \\
& \vec{j}=n Q \overrightarrow{v_{d}}=\frac{\vec{E}}{\rho} \\
& R_{e q}=R_{1}+R_{2} \text { (In series) } \\
& \frac{1}{R_{e q}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}(\text { In parallel }) \\
& \sum_{\text {junction }} I=0 \\
& \sum_{\text {loop }} V=0 \\
& \vec{\nabla} f=\frac{\partial f}{\partial r} \hat{r}+\frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta}+\frac{\partial f}{\partial z} \hat{z} \\
& d \vec{l}=d r \hat{r}+r d \theta \hat{\theta}+d z \hat{z} \\
& \text { (Cylindrical Coordinates) } \\
& \vec{\nabla} f=\frac{\partial f}{\partial r} \hat{r}+\frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta}+\frac{1}{r \sin (\theta)} \frac{\partial f}{\partial \phi} \hat{\phi} \\
& d \vec{l}=d r \hat{r}+r d \theta \hat{\theta}+r \sin (\theta) d \phi \hat{\phi} \\
& \text { (Spherical Coordinates) } \\
& y(t)=\frac{B}{A}\left(1-e^{-A t}\right)+y(0) e^{-A t} \\
& \text { solves } \frac{d y}{d t}=-A y+B \\
& y(t)=y_{\max } \cos (\sqrt{A} t+\delta) \\
& \text { solves } \frac{d^{2} y}{d t^{2}}=-A y \\
& \int_{0}^{\infty} x^{n} e^{-a x} d x=\frac{n!}{a^{n+1}} \\
& \int_{0}^{\infty} x^{2 n} e^{-a x^{2}} d x=\frac{(2 n)!}{n!2^{2 n+1}} \sqrt{\frac{\pi}{a^{2 n+1}}} \\
& \int_{0}^{\infty} x^{2 n+1} e^{-a x^{2}} d x=\frac{n!}{2 a^{n+1}} \\
& \int\left(1+x^{2}\right)^{-1 / 2} d x=\ln \left(x+\sqrt{1+x^{2}}\right) \\
& \int\left(1+x^{2}\right)^{-1} d x=\arctan (x) \\
& \int\left(1+x^{2}\right)^{-3 / 2} d x=\frac{x}{\sqrt{1+x^{2}}} \\
& \int \frac{x}{1+x^{2}} d x=\frac{1}{2} \ln \left(1+x^{2}\right) \\
& \int \frac{1}{\cos (x)} d x=\ln \left(\tan \left(\frac{x}{2}+\frac{\pi}{4}\right)\right) \\
& \int \frac{x}{(1+x)^{3 / 2}} d x=\frac{2(x+2)}{\sqrt{1+x}} \\
& \frac{d \cot (x)}{d x}=-\csc ^{2}(x) \\
& \sin (x) \approx x \\
& \cos (x) \approx 1-\frac{x^{2}}{2} \\
& e^{x} \approx 1+x+\frac{x^{2}}{2} \\
& (1+x)^{\alpha} \approx 1+\alpha x+\frac{(\alpha-1) \alpha}{2} x^{2} \\
& \ln (1+x) \approx x-\frac{x^{2}}{2} \\
& \sin (2 x)=2 \sin (x) \cos (x) \\
& \cos (2 x)=2 \cos ^{2}(x)-1 \\
& \sin (a+b)=\sin (a) \cos (b)+\cos (a) \sin (b) \\
& \cos (a+b)=\cos (a) \cos (b)-\sin (a) \sin (b) \\
& 1+\cot ^{2}(x)=\csc ^{2}(x) \\
& 1+\tan ^{2}(x)=\sec ^{2}(x)
\end{aligned}
$$

## MT2- E. Corsini - Physics 7B - UC Berkeley

In In each problem express your answer in terms of known or given variables listed for that problem
In addition to the known variables all physical constants are also known $\left(\boldsymbol{g}, \boldsymbol{\varepsilon}_{0}, \mathbf{G}, \ldots\right)$
Not all variables need to be used in your answers
Show your work, box your answers, check units.
Problem 1(total: 20 points)
The known variables are $V R_{1} R_{2} C_{1} C_{2}$
Consider the circuit as shown.
Before $t=0$ both switches are open and the capacitors are uncharged.
At time $t=0$ we close switch 1 and switch 2 remains open,
a) At time $t=0$ what are the currents $I 1$ and $I 2$ in each arm?
b) At time $t=0$ what is the potential difference between point $a$ and point $b$ ?
c) At time $t=00$ what is the potential difference between point a and point $b$ ? Now consider the circuit in the state as it would be in part c). We close switch 2 and both switches are now closed
d) What is the final potential of point $b$ with respect to ground at $t=00$ ?
e) How much does the charge change on each capacitor at $t=00$ (compared to just before switch 2 was closed)?

Problem 2 (total: 20 points) The known variables are: ba $V_{a b} \rho$
The region between two concentric spherical conducting thin shells of radii a and $\boldsymbol{b}(\boldsymbol{b}>\boldsymbol{a})$ is filled with a material with resistivity $\boldsymbol{\rho}$. The potential difference between the two shells is $V_{a b}$
a) Derive an expression for the total resistance $R$ between the two shells?
b) Show that the expression from a) reduces to an expression similar to $R=\rho L / A$ when the separation $L=b-a$ between the spheres is small.
c) What is the current density $\vec{j}$ as a function of $\boldsymbol{r}(\boldsymbol{a}<\boldsymbol{r}<\boldsymbol{b})$; in this part $\boldsymbol{r}$ is given.

Problem 3 (total: $\mathbf{2 0}$ points) The known variables are $\boldsymbol{K} \boldsymbol{L} \boldsymbol{h} \boldsymbol{d} \boldsymbol{\xi}$ (pronounced xi) A fuel gauge uses the value of the capacitance $\boldsymbol{C}$ to measure the fuel level in the tank. Each of the capacitor's plates have a size $L x L$; the separation between the plates is $\boldsymbol{d}(L \gg \boldsymbol{d})$. The height of the fuel between the plates is $\boldsymbol{h}$. The fuel dielectric constant is $K$
a) What is the capacitance $\boldsymbol{C}$ as a function of $\boldsymbol{h}$

The fuel gauge is calibrated for a capacitance value when the dielectric constant value of the fuel is $K$
Suppose you now replace the fuel in part a) with the same volume of a different fuel with dielectric constant $(K+\xi)$, where $\xi \ll K$
b) What is \% the error on the fuel gauge, with this new fuel ?

## Problem 4 (total: 20 points)

The known variables are $d q$
Consider the ionic crystal as shown (all sides have length $\boldsymbol{d}$ ) with six charges of equal magnitudes $\boldsymbol{q}$, one at each apex, with charge signs arranged as shown. Take zero to be the potential energy of the six charges when they are infinitely far from each other.
a) Calculate the potential energy $\boldsymbol{U}$ of this arrangement.
b) Is the ionic crystal stable? Explain why.

Problem 5 (total: $\mathbf{2 0}$ points): The known variables are: $\operatorname{Rrd\rho }$
An insulating cylinder of radius $R$ has a symmetry axis through the origin. The cylinder has a cylindrical cavity of radius $r$ with axis of symmetry parallel to that of the cylinder and at a distance $\boldsymbol{d}$ from the origin, as shown ( $r<\boldsymbol{d}<\boldsymbol{R}-r$ ). The solid part of the cylinder has uniform charge density $\rho$
Find the magnitude and direction of the electric field inside the cylindrical cavity. Use either the Cartesian coordinates $\mathbf{x}, \mathbf{y}$ or the polar coordinates $\mathbf{r}, \boldsymbol{\theta}$. Both coordinate systems have the same origin $\mathbf{o}$


