Physics 7A Lecture 2 Fall 2014
Final Solutions

December 22, 2014

PROBLEM 1

The string is oscillating in a transverse manner. The wave velocity of the string is thus

$$v = \sqrt{\frac{T_s}{\mu}},$$

where T_s is tension and μ is the linear mass density. We can relate the wavelength λ of the string to the angular frequency ω of the string like so:

$$\omega = 2\pi \frac{v}{\lambda}.$$

The vertical acceleration of the string given amplitude A_{min} is

$$|a| = \omega^2 A_{min}.$$

The bug will just lose contact when the string is accelerating more than gravity, which is the condition that $|a| = g$. We therefore get that

$$g = \omega^2 A_{min} \implies A_{min} = \frac{g}{\omega^2} = \frac{g\lambda^2}{4\pi^2 v^2} = \frac{g\lambda^2 \mu}{4\pi^2 T_s}.$$

Problem 2

Part (a)

Since the rod is hinged in the center, the torque due to gravity is 0
Let the extension of the spring be x

For very small angles θ

$$x = \frac{l}{2}\sin\theta \tag{1}$$
$$x \sim \frac{l}{2}\theta \tag{2}$$
$$\tag{3}$$

Thus, the torque is

$$\tau = kx\frac{l}{2}\cos\theta \tag{4}$$
$$\tau \sim \frac{kl^2\theta}{4} \tag{5}$$
$$\tag{6}$$

Thus, writing torque-acceleration equation gives

$$-\frac{1}{12}ml^2\alpha = \tau \tag{7}$$
$$-\frac{1}{12}ml^2\alpha = \frac{kl^2\theta}{4} \tag{8}$$
$$\alpha = -\frac{3k}{m}\theta \tag{9}$$
$$\tag{10}$$

Comparing with $\alpha = -\omega^2\theta$

$$\omega = \sqrt{\frac{3k}{m}} \tag{11}$$

Part (b)

After the rod is cut, mass $m \to m/2$ and length $l \to l/2$. Also, the gravity acts as the center of mass and provides torque.The position of the center of mass is $l/4$ from the hinge.
For very small angles θ, the torque becomes

$$\tau = \frac{m}{2} g \frac{l}{4} \sin\theta \tag{12}$$

$$\tau = \frac{mgl}{8}\theta \tag{13}$$

$$\tag{14}$$

Total torque thus becomes

$$\tau = \frac{mgl}{8}\theta + \frac{kl^2}{4}\theta \tag{15}$$

$$\tag{16}$$

The moment of inertia is

$$I = \frac{1}{3}\frac{m}{2}\frac{l^2}{2^2} \tag{17}$$

$$\tag{18}$$

$$-\frac{1}{24}ml^2\alpha = \tau \tag{19}$$

$$-\frac{1}{24}ml^2\alpha = \frac{mgl}{8}\theta + \frac{kl^2}{4}\theta \tag{20}$$

$$\alpha = -\frac{(\frac{mgl}{8} + \frac{kl^2}{4})}{\frac{1}{24}ml^2}\theta \tag{21}$$

$$\tag{22}$$

Comparing with $\alpha = -\omega^2\theta$

$$\omega = \sqrt{\frac{6k}{m} + \frac{3g}{l}} \tag{23}$$

Problem 3 Solution

Physics 7A Section 2 Final (Hallatschek)

December 2014

The power required to run conveyor is given by $F_{net}v$, where F_{net} is the net force the conveyor belt must exert upward along the ramp (parallel to surface of ramp) and v is the constant velocity of the belt itself. The terms that make up F_{net} come from three sources:

Force from inelastic collision on ramp:

The falling mass is continuously being accelerated up to the speed v as it hits the ramp. The external force required to sustain this acceleration of mass flow is given by

$$\frac{\Delta P}{\Delta t} = \frac{\rho \Delta t v}{\Delta t} = \rho v.$$

Force to hold up mass already on ramp against gravity:

This force is the usual $mg \sin\theta$, and since the mass on the ramp at time Δt is given by $\rho \Delta t$, we have for this force

$$\rho \Delta t g \sin\theta.$$

Force to counteract friction force F_{Fr}:

This force is simply F_{Fr}.

Summing the above three forces and multiplying by v to get the power, we get

$$Power = (\rho v + \rho \Delta t g \sin\theta + F_{Fr})\, v \tag{1}$$

Note on solutions:

Some students attempted to calculate the power by calculating the change in energy as a function of time, where energy includes both potential and kinetic energy of the mass on conveyor. This method ignores the energy that is lost due to the inelastic collision with the conveyor belt, but is otherwise correct.

Physics 7A Section 2 Prof. Hallatschek Problem 4

Trevor Grand Pre

December 20, 2014

(a)To solve this problem we set up te force equation of this system.

$$F = \frac{GM_E m}{r^2} = M_E a_E$$

Solving for the acceleration, we get

$$a_E = \frac{mG}{r^2}.$$

(b) From this, we can see that the the acceleraton for the closest and farthest point on earth is $a_{far} = \frac{mG}{(r+r_E)^2}$ and $a_{near} = \frac{mG}{(r-r_E)^2}$.

When we subtract the two and then simplify, we get

$$a_{near} - a_{far} = \frac{mG}{(r-r_E)^2} - \frac{mG}{(r+r_E)^2} = \frac{4mGr_E r}{(r^2 - r_E{}^2)^2}$$

Degenkolb

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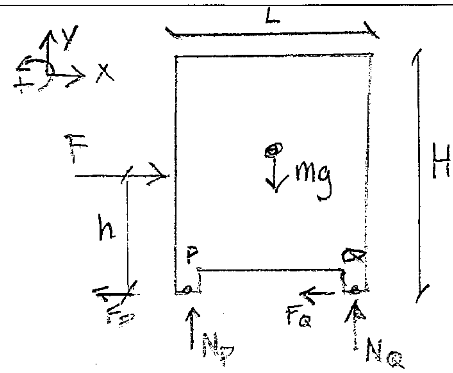
PROBLEM 5 - SOLUTION



A). Find "F" if dresser is sliding at a constant velocity.

Constant velocity → $a_x = 0$
Sliding → $a_y = 0$

$\Sigma F_x = 0$

$F - F_P - F_Q = 0$

$F = \mu_K (N_P + N_Q)$

$\Sigma F_y = 0$

$N_P + N_Q - mg = 0$

$N_P + N_Q = mg$

$$\boxed{F = \mu_K (mg)}$$

B). Find magnitudes of N_P & N_Q (in m, g, L, h, μ_K)

$\Sigma T_Q = 0$

$-Fh - N_P L + mg \frac{L}{2} = 0$

$N_P = \left(\frac{mgL}{2} - Fh\right) \frac{1}{L}$

$$\boxed{N_P = mg\left(\frac{1}{2} - \mu_K \frac{h}{L}\right)}$$

$\Sigma F_y = 0$

$N_P + N_Q = mg$

$$\boxed{N_Q = mg\left(\frac{1}{2} + \mu_K \frac{h}{L}\right)}$$

C). FIND h_{max} so dresser will not topple

Begins to topple when $N_P \to 0$

$\Sigma T_Q = 0$

$-Fh + mg \frac{L}{2} = 0$

$h = mg \frac{L}{2} \frac{1}{F}$

$$\boxed{h_{max} = \frac{L}{2\mu_K}}$$

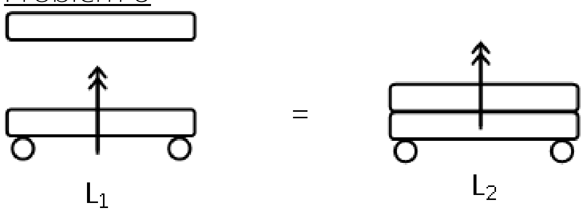
ALTERNATE

$N_P = mg\left(\frac{1}{2} - \mu_K \frac{h}{L}\right) = 0$

$\mu_K \frac{h}{L} = \frac{1}{2}$

$h = \frac{L}{2\mu_K}$

Problem 6



Part A

Sum vectors about y-Axis

$$I_t w_i = (I_t + I_r) w_f$$

$$w_f = \frac{w_i I_t}{I_t + I_r} \qquad \text{ANS}$$

Part B

Initial Kinetic Energy

$$K_i = \frac{1}{2} I_t w_i^2$$

$$w_i^2 = \frac{2K_i}{I_t} \qquad (1)$$

Final Kinetic Energy

$$K_f = \frac{1}{2}(I_t + I_r) w_f^2$$

Sub from Part A

$$K_f = \frac{1}{2}(I_t + I_r)\left(\frac{w_i I_t}{I_t + I_r}\right)^2$$

$$K_f = \frac{1}{2}(I_t + I_r)\left(\frac{w_i I_t}{I_t + I_r}\right)^2$$

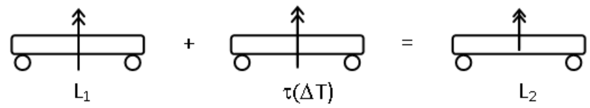
$$K_f = \frac{1}{2}(I_t + I_r)\left(\frac{w_i I_t}{I_t + I_r}\right)^2$$

$$K_f = \frac{(I_t + I_r) w_i^2 I_t^2}{2(I_t + I_r)^2}$$

Sub from Equation (1)

$$K_f = \frac{K_i I_t}{(I_t + I_r)} \qquad \text{ANS}$$

Part C



Sum vectors about y-Axis

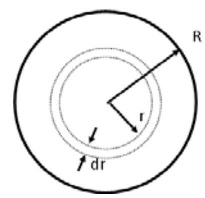
$$L_1 + J = L_2$$

$$I_t w_i + \tau(\Delta t) = I_t w_f$$

$$\tau = \frac{I_t(w_f - w_i)}{\Delta t} \qquad (2)$$

ANS

Part D



Friction acting on ring

$$df = \mu(dN)$$

$$= \mu g \left(\frac{m_r}{\pi R^2}\right) 2\pi r dr$$

$$= \frac{2\mu g m_r}{R^2} r dr$$

Torque due to "friction ring"

$$\tau = \int r df$$

$$\tau = \int_0^R \frac{2\mu g m_r}{R^2} r^2 dr$$

$$= \frac{2}{3}\mu g m_r R$$

$$= \frac{4\mu g I_r}{3R}$$

Sub into (2)

$$\frac{4\mu g I_r}{3R} = \frac{I_t(w_f - w_i)}{\Delta t}$$

$$\Delta t = \frac{3R I_t(w_f - w_i)}{4\mu g I_r} \qquad (2)$$

ANS

Lecture 2 Final
Problem 7 Solution

(a):
Consider a differential volume element of water. Newton's second law tells us:

$$P_{outer}dA - P_{inner}dA = \rho\, dA\, dr\, \omega^2\, r$$

The dA from each term cancel. $P_{outer} - P_{inner}$ becomes dP, since it is a difference in pressure across a differentially small volume element.

$$dP = \rho\, dr\, \omega^2\, r$$

Integrating both sides gives us:

$$P - P_0 = \tfrac{1}{2}\, \rho\, \omega^2\, (r^2 - r_0^2)$$
$$P = P_0 + \tfrac{1}{2}\, \rho\, \omega^2\, (r^2 - r_0^2)$$

(b):
Use Bernoulli's principle. Equate a point just inside the test tube to a point just outside. Realize that a point just inside will have the pressure found in part (a) with r = L. A point just outside will have a pressure P_0 (since it is exposed to the atmosphere). Therefore:

$$P_0 + \tfrac{1}{2}\, (\rho\, v^2) = P_0 + \tfrac{1}{2}\, \rho\, \omega^2\, (L^2 - r_0^2)$$
$$v = \omega\, \mathrm{Sqrt}[L^2 - r_0^2]$$

(c):
$A_1v_1 = A_2v_2$, so (a v / A) = dx/dt. Solve for dt, then integrate. The bounds on the dt side are 0 to t. The bounds on the dx side are r_0 to L.

$$dt = (A/a)\, dx\, (1 / \omega\, \mathrm{Sqrt}[L^2 - x^2])$$
$$t = (A/ \omega\, a)\, \mathrm{Cos}^{-1}\, (r_0 / L)$$