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Friday, March 1, 2013, 2-3 PM.
Please write your name at the top of each page as indicated and write all answers in the space provided. If you need additional space, write on the back sides. Do not remove or add any pages. Assume all problems are two-dimensional unless noted otherwise. For all answers, where appropriate, provide units. Good luck!

## PROBLEM 1: 30 pts total

For this 3D problem, a force P of magnitude 520 lb . acts at point E on the frame, as shown.

a- Determine the $x-y-z$ components of the force P. (10 pts)
b- Find the moment of the force $P$ about point $D$. Express your answer as a vector. ( $\mathbf{1 0} \mathbf{~ p t s ) ~}$
c- Find the magnitude of the component of this moment that causes rotation about an axis along C-D. (10 pts)
$\qquad$

## PROBLEM 2: 35 pts total

Rod $A B$ is supported by a frictionless hinge joint at $A$ and rests without moving against a frictionless circular supporting peg at C. A vertical force of 170 N is applied at B . In your analysis, ignore the mass of the rod.
a- Draw a fully labeled free body diagram of the rod. (10 pts)

b- Using equilibrium equations, determine the magnitude of the contact force at C . (10pts)
c- Consider now a situation in which the supporting peg can be moved to other locations along the rod.
Using a graphical approach, find the location of the peg that minimizes the magnitude of the reaction force at $A$. (Hint: What is the direction of that minimum force?) (15 pts)

## PROBLEM 3: 35 pts total

Beam abcd of length 3L has external loads and boundary conditions as shown: End moment $M_{a}$, End force $F_{d}$, distributed loading varying from $W_{0} N / m$ at $b$ to $W_{1} N / m$ at $c$, hinge joint at point $a$, and roller at point c. Ignore the mass of the beam. Assume $\mathrm{x}=0$ at point a.

a- Determine the magnitude and x -location of the resultant force $\mathbf{R}$ for the distributed loading. (15 pts)

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b- The above free-body diagram is to be replaced by the one shown below, in which $F_{E Q}$ is statically equivalent to the external loads shown above. Using the numerical values of the external loads as shown below, find the magnitude of the force $F_{E Q}$ and its location, $x$. (20 pts)

Assume $\mathrm{M}_{\mathrm{a}}=175 \mathrm{Nm}, \mathrm{F}_{\mathrm{d}}=100 \mathrm{~N}$, $\mathrm{W}_{0}=150 \mathrm{~N} / \mathrm{m}, \mathrm{W}_{1}=50 \mathrm{~N} / \mathrm{m}$ and $\mathrm{L}=3 \mathrm{~m}$


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## Formula Sheet

## Sine Rule and Cosine Rule



$$
\begin{aligned}
& \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
& c^{2}=a^{2}+b^{2}-2 a b \cos C \\
& b^{2}=a^{2}+c^{2}-2 a c \cos B \\
& a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{aligned}
$$

## Vector Algebra Formulae

$\mathbf{V}=\mathbf{P} \times \mathbf{Q}=\left(P_{x} \mathbf{i}+P_{y} \mathbf{j}+P_{z} \mathbf{k}\right) \times\left(Q_{x} \mathbf{i}+Q_{y} \mathbf{j}+Q_{z} \mathbf{k}\right)=\mathbf{V}=\left|\begin{array}{ccc}\mathbf{i} & \mathbf{j} & \mathbf{k} \\ P_{x} & P_{y} & P_{z} \\ Q_{x} & Q_{y} & Q_{z}\end{array}\right|$
$V=P Q \sin \theta$ $\mathbf{P} \cdot \mathbf{Q}=P Q \cos \theta$

$$
\begin{aligned}
& V_{x}=P_{y} Q_{z}-P_{z} Q_{y} \\
& V_{y}=P_{z} Q_{x}-P_{x} Q_{z} \\
& V_{z}=P_{x} Q_{y}-P_{y} Q_{x}
\end{aligned}
$$

Moment

$$
\mathbf{M}_{O}=\mathbf{r} \times \mathbf{F} \quad M_{O L}=\boldsymbol{\lambda} \cdot \mathbf{M}_{O}=\boldsymbol{\lambda} \cdot(\mathbf{r} \times \mathbf{F})
$$

## Centroid and Center of Gravity

$$
\left.\begin{array}{lrl}
\bar{x} A=\int x d A & \bar{y} A=\int y d A & \bar{x} W \\
\bar{X} \Sigma A & =\Sigma \bar{x} A & \bar{X} \Sigma W \\
\bar{X} & =\Sigma \bar{x} W \\
\bar{Y} \Sigma A & =\Sigma \bar{y} A & \bar{Y} \Sigma W
\end{array}\right) \Sigma \bar{y} W=\int y d W
$$

| Shape |  | $\bar{x}$ | $\bar{y}$ | Area |
| :---: | :---: | :---: | :---: | :---: |
| Triangular area |  |  | $\frac{h}{3}$ | $\frac{b h}{2}$ |
| Quarter-circular area |  | $\frac{4 r}{3 \pi}$ | $\frac{4 r}{3 \pi}$ | $\frac{\pi r^{2}}{4}$ |
| Semicircular area |  | 0 | $\frac{4 r}{3 \pi}$ | $\frac{\pi r^{2}}{2}$ |
| Quarter-elliptical area |  | $\frac{4 a}{3 \pi}$ | $\frac{4 b}{3 \pi}$ | $\frac{\pi a b}{4}$ |
| Semielliptical area |  | 0 | $\frac{4 b}{3 \pi}$ | $\frac{\pi a b}{2}$ |

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