You have 2 hours to complete the exam.

The exam is closed book, closed notes, closed computer, closed calculator, except one hand-written 8.5” × 11” crib sheet of your own creation and the official 61A midterm 1 study guide attached to the back of this exam.

Mark your answers ON THE EXAM ITSELF. If you are not sure of your answer you may wish to provide a brief explanation.

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All the work on this exam is my own. (please sign)

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<table>
<thead>
<tr>
<th>Q. 1</th>
<th>Q. 2</th>
<th>Q. 3</th>
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<tr>
<td>/12</td>
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1. (12 points) In-N-Out

For each of the expressions in the tables below, write the output displayed by the interactive Python interpreter when the expression is evaluated. The output may have multiple lines. No errors occur.

The first two rows have been provided as examples of the behavior of the built-in `pow` and `print` functions.

*Recall:* The interactive interpreter displays the value of a successfully evaluated expression, unless it is `None`.

Assume that you have started Python 3 and executed the following statements:

```python
from operator import add

def re(peat):
    return print(peat, peat)

def cheap(eat):
    car, seat = re, print
    seat(car(eat))
    return double(eat)

def double(double):
    if double:
        return double + double
    elif car(double)(print)(print):
        return 1000
    else:
        return seat(3)

seat = double
car = lambda c: lambda a: lambda r: r(5, a(c))
```

<table>
<thead>
<tr>
<th>Expression</th>
<th>Interactive Output</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>pow(2, 3)</code></td>
<td>8</td>
</tr>
<tr>
<td><code>print(4, 5)</code></td>
<td>4 5</td>
</tr>
<tr>
<td><code>print(re(1+2), print(4))</code></td>
<td>3 3 4 None None</td>
</tr>
<tr>
<td><code>cheap(3)</code></td>
<td>3 3 None 6</td>
</tr>
<tr>
<td><code>cheap(seat(2))</code></td>
<td>4 4 None 8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expression</th>
<th>Interactive Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>car(1)(double)(pow)</td>
<td>25</td>
</tr>
<tr>
<td>double(print(1))</td>
<td>1 5 None 6</td>
</tr>
<tr>
<td>car(0)(seat)(add)</td>
<td>0 5 None 11</td>
</tr>
</tbody>
</table>
2. (14 points) Supernatural

(a) (6 pt) Fill in the environment diagram that results from executing the code below until the entire program is finished, an error occurs, or all frames are filled. You may not need to use all of the spaces or frames.

A complete answer will:

- Add all missing names and parent annotations to all local frames.
- Add all missing values created or referenced during execution.
- Show the return value for each local frame.

Remember: Do not add a new frame when calling a built-in function (such as \(\text{abs}\)).

```python
def nanana(batman):
    while batman(superman) > ivy:
        def batman(joker):
            return ivy
        return -ivy
    return nanana

def joker(superman):
    if superman(batman):
        ivy = -batman
        return nanana
    joker(abs)(abs)
```

Global frame

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<tr>
<th></th>
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<tbody>
<tr>
<td>batman</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>superman</td>
<td>-2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ivy</td>
<td>-3</td>
<td></td>
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<tr>
<td>joker</td>
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func joker(superman) [parent=Global]

func nanana(batman) [parent=Global]

func abs(...) [parent=Global]

f1: joker [parent=Global]

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<tbody>
<tr>
<td>superman</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>ivy</td>
<td>-1</td>
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Return Value

f2: nanana [parent=Global]

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<tbody>
<tr>
<td>batman</td>
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</table>

func batman(joker) [parent=f2]

f3: batman [parent=f2]

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</thead>
<tbody>
<tr>
<td>joker</td>
<td>-2</td>
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</table>

Return Value

"
(b) (8 pt) Fill in the environment diagram that results from executing the code below until the entire program is finished, an error occurs, or all frames are filled. You may not need to use all of the spaces or frames.

A complete answer will:
- Add all missing names and parent annotations to all local frames.
- Add all missing values created or referenced during execution.
- Show the return value for each local frame.

```
def still(glad):
    def heart(broken):
        glad = lambda heart: lambda: heart-broken
        return glad(grin)
    return heart(glad-grin)()

broken, grin = 5, 3
still(broken-1)
```
3. (14 points) You Complete Me

(a) (4 pt) Implement the `longest_increasing_suffix` function, which returns the longest suffix (end) of a positive integer that consists of strictly increasing digits.

```python
def longest_increasing_suffix(n):
    """Return the longest increasing suffix of a positive integer n.
    >>> longest_increasing_suffix(63134)
    134
    >>> longest_increasing_suffix(233)
    3
    >>> longest_increasing_suffix(5689)
    5689
    >>> longest_increasing_suffix(568901) # 01 is the suffix, displayed as 1
    1
    """
    m, suffix, k = 10, 0, 1
    while n:
        n, last = n // 10, n % 10
        if last < m:
            m, suffix, k = last, suffix + k * last, 10 * k
        else:
            return suffix
    return suffix
```

(b) (3 pt) Add parentheses and single-digit integers in the blanks below so that the expression on the second line evaluates to 2015. **You may only add parentheses and single-digit integers.** You may leave some blanks empty.

```python
lamb = lambda lamb: lambda: lamb + lamb

lamb(1000)(1)(lambda b, c: b()*(b()-c)(lamb(2), 1))
```
(c) (3 pt) Implement the `combine` function, which takes a non-negative integer `n`, a two-argument function `f`, and a number `result`. It applies `f` to the first digit of `n` and the result of combining the rest of the digits of `n` by repeatedly applying `f` (see the doctests). If `n` has no digits (because it is zero), `combine` returns `result`.

```python
from operator import add, mul

def combine(n, f, result):
    """Combine the digits in non-negative integer n using f.
    >>> combine(3, mul, 2) # mul(3, 2)
    6
    >>> combine(43, mul, 2) # mul(4, mul(3, 2))
    24
    >>> combine(6502, add, 3) # add(6, add(5, add(0, add(2, 3)))
    16
    >>> combine(239, pow, 0) # pow(2, pow(3, pow(9, 0)))
    8
    ""
    if n == 0:
        return result
    else:
        return combine(n // 10, f, f(n % 10, result))
```

(d) (4 pt) Implement the `memory` function, which takes a number `x` and a single-argument function `f`. It returns a function with a peculiar behavior that you must discover from the doctests. You may only use names and call expressions in your solution. You may not write numbers or use features of Python not yet covered in the course.

```python
square = lambda x: x * x
double = lambda x: 2 * x

def memory(x, f):
    """Return a higher-order function that prints its memories.
    >>> f = memory(3, lambda x: x)
    >>> f = f(square)
    3
    >>> f = f(double)
    9
    >>> f = f(print)
    6
    >>> f = f(square)
    3
    None
    ""
    def g(h):
        print(f(x))

        return memory(x, h)
    return g
```