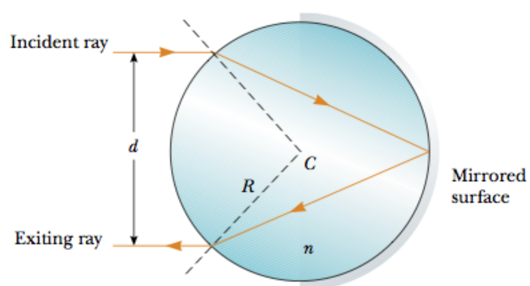
PHYSICS 7C FALL 2014 MIDTERM 1

All of the problems are worth 10 points. To receive full credit you must show **all** of your work, however, you do not need to derive any formulas that appear on this exam, or that you can recall from memory.

(1) The electric field of a plane wave in free space is $\mathbf{E} = A(y + ct)\hat{\mathbf{z}}$, where A is a constant. What is the magnetic field?

(2) A spherical particle of radius r and mass density ρ is located a distance $R >> r$ away from the Sun (the Sun has mass M). If the intensity of the EM radiation from the sun is I at the location of the particle, for what value of r does the force from the EM radiation exactly cancel out the force of gravitational attraction? [Recall that the gravitational force between two masses, m_1 and m_2, separated by a distance r_{12}, is Gm_1m_2/r_{12}^2]

(3) A transparent cylinder of radius R has a mirrored surface on its right half as shown below.



If an incident ray enters the cylinder, reflects off the back wall, and emerges antiparallel to the incident ray, a distance d away from it, what is the index of refraction of the cylinder in terms of R and d? Assume that the index of refraction of the surrounding air is 1. You may leave your answer in terms of inverse sine, or make use of the identity $\sin\left(\frac{x}{2}\right) = \sqrt{\frac{1-\cos x}{2}}$.

(4) A goldfish is swimming at a speed v normal to the the front wall of a rectangular aquarium. What is the apparent speed of the fish measured by an observer looking through the front wall at normal incidence? Assume that the water and the glass tank have an index of refraction n, and that the index of refraction of air is 1.

(5) In many applications it is necessary to expand or to decrease the diameter of a beam of parallel rays of light. This change can be made by using a diverging lens, followed by a converging one. If diverging lens has focal length $-f_1 < 0$, and the converging lens has focal length $f_2 > f_1$, by what factor can you increase the diameter of a beam of parallel rays? [Hint: a ray diagram is quite useful in this case.]

Solutions

(1) Even though this E-field is not sinusoidal, it is still a plane wave because it is constant over planes perpendicular to the y axis. Thus, we know that $E = cB$, $\mathbf{E}$ and $\mathbf{B}$ are perpendicular, and the wave travels in the direction of $\mathbf{E} \times \mathbf{B}$. Since the wave is traveling in the $-\hat{\mathbf{y}}$ direction, the B-field must be

$$\mathbf{B} = -\frac{A}{c}(y + ct)\hat{\mathbf{x}}$$

This can be confirmed by plugging this solution into Maxwell's Equations in free space

$$\nabla \cdot \mathbf{E} = \nabla \cdot \mathbf{B} = 0,\ \nabla \times \mathbf{E} = \frac{\partial E_y}{\partial y}\hat{\mathbf{x}} = A\hat{\mathbf{x}} = -\frac{\partial \mathbf{B}}{\partial t},\ \nabla \times \mathbf{B} = -\frac{\partial B_x}{\partial y}\hat{\mathbf{z}} = \frac{A}{c}\hat{\mathbf{z}} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

This solution could also have been found by solving Maxwell's Equations in free space directly.

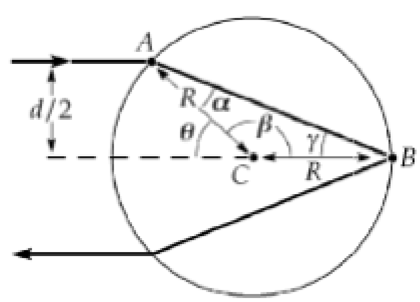
(2) The force due to the radiation pressure is

$$F_{\text{rad}} = \int_H \frac{\mathbf{I}}{c} \cdot \mathbf{dA} = \frac{I}{c}\pi r^2$$

where H is the hemisphere of particle that is hit by the radiation. Note that the effective area to be multiplied by $\frac{I}{c}$ is the projection of the hemisphere onto the plane perpendicular to the direction in which the radiation is moving, πr^2. Setting this equal to the gravitational force we have

$$\frac{I}{c}\pi r^2 = \frac{GM\rho\frac{4}{3}\pi r^3}{R^2} \Rightarrow r = \frac{3R^2 I}{4GM\rho c}$$

(3) Let us label our angles as depicted below



Clearly the triangle ABC is isosceles, hence $\alpha = \gamma$. $\beta = 180 - \theta$, hence

$$180 - \theta = \beta = 180 - \alpha - \gamma = 180 - 2\alpha \Rightarrow \alpha = \frac{\theta}{2}$$

Looking at the picture we see immediately that $\sin\theta = \frac{d}{2R}$. From Snell's Law we have

$$\frac{d}{2R} = \sin\theta = n\sin\alpha = n\sin\left[\frac{1}{2}\sin^{-1}\left(\frac{d}{2R}\right)\right] \Rightarrow n = \frac{d}{2R\sin\left[\frac{1}{2}\sin^{-1}\left(\frac{d}{2R}\right)\right]}$$

Or, using the identity provided, we have

$$\frac{d}{2R} = \sin\theta = n\sin\alpha = n\sin\left(\frac{\theta}{2}\right) = n\sqrt{\frac{1-\cos\theta}{2}} \Rightarrow n = \frac{d}{2R}\sqrt{\frac{2}{1-\sqrt{1-\frac{d^2}{4R^2}}}}$$

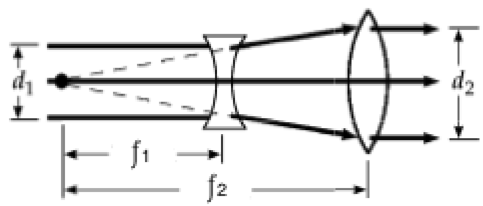
(4) This is just velocity of the image of an object at position $d_o = x_0 - vt$ behind a spherical boundary of infinite radius.

$$\frac{1}{d_i} = \frac{n-1}{\infty} - \frac{n}{d_o} \Rightarrow d_i = -\frac{d_o}{n}$$

Thus, the apparent speed of the fish is

$$v_{\text{image}} = \frac{dd_i}{dt} = -\frac{1}{n}\frac{dd_o}{dt} = \frac{v}{n}$$

(5) Assuming that the light is coming from the left, the parallel rays that hit the diverging lens will diverge and appear to be coming from a point a distance f_1 to the left of the diverging lens. In order for these rays to become parallel again after passing through the converging lens they must appear to be coming from an object a distance f_2 away from the diverging lens. See the picture below



Using similar triangles we have

$$\frac{d_2}{d_1} = \frac{f_2}{f_1} \Rightarrow d_2 = \left(\frac{f_2}{f_1}\right) d_1$$

Hence, we can increase the diameter by a factor of f_2/f_1.