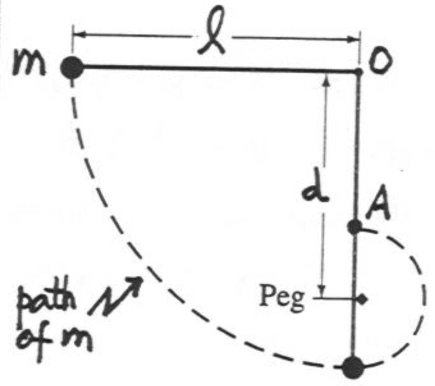
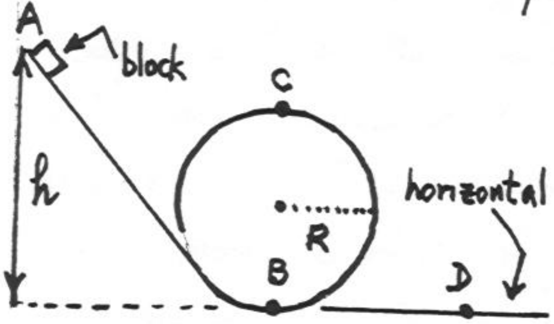
Physics 7A (Sec 2) Midterm Exam #2 Nov. 5, 2002

You may use two (2) cards, 3" x 5", as memory aids. Exam = 200 points

(30) (1) A small particle of mass m is held horizontal at the end of a massless string of length l, as shown. A peg is located at a distance d vertically below point O where the string is attached. Mass m is released, the string catches on the peg (when the string is vertical and the mass m describes a new circular path with the peg as its center. In order to complete this new circular path, the tension in the string must be non-zero when the mass reaches point A. There exists a critical value d_c of the distance d for which the tension in the string is zero at point A. Calculate the value of d_c in terms of l.

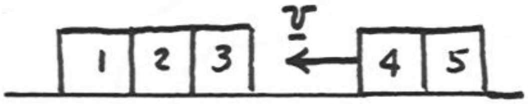


(30) (2) Given a "loop-the-loop" as shown, with a circular loop of radius R. A small block of mass m slides (without friction, starting from rest at point A. (a) Calculate the minimum value of h for which the block will remain on the track; (b) If N_C is the magnitude of the normal force exerted by the track on the block at point C, and N_B is the magnitude of the normal force exerted on the block at point B, calculate $(N_B - N_C)$ in terms of m and g; (c) Calculate the speed v_D of the block at point D; (d) Does $(N_B - N_C)$ depend on h and/or R? Justify your answer. [Part (a) = 10, (b) = 10, (c) = 5, (d) = 5 points]



(continued ⟶)

(30) (3) Three identical blocks (numbered 1, 2, 3), each of mass m, are at rest and in contact on a frictionless horizontal surface, as shown. Two identical blocks (4, 5) are also identical to blocks 1, 2, 3 and are in contact.

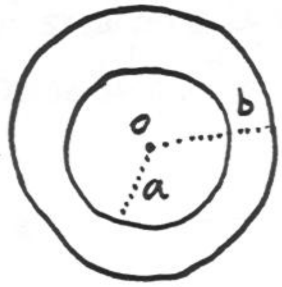


Blocks 4 and 5 move with speed v, collide elastically with block 3 (which remains at rest throughout); blocks 4 and 5 come to rest after the collision. Prove that, after the collision, blocks 1 and 2 (still in contact) move off to the left with speed v' instead of block 1 moving off with speed $(2v')$.

(30) (4) Given a symmetric three-dimensional rigid body (for example, a steel sphere) at rest at the top of an incline so that the center of mass of the body is at a height h above the bottom of the incline. The body rolls down the incline without slipping. (a) Show that, at the bottom of the incline, the speed v_{cm} of the center of mass of the body is independent of the mass and dimensions of the body; (b) Describe a possible experiment to measure (using your answer to (a)) the numerical pre-factor in the expression for the moment of inertia (relative to the center of mass) of a rigid body of arbitrary geometric shape. [(a) = 25, (b) = 5 pts]

(continued →)

(40) (5) Given a thin solid circular annular ring of inner radius a and outer radius b ($b > a$), as shown. The area density of the material of the ring is σ kg m^{-2}. Consider a point mass m at a vertical distance z above the plane of the ring and on the axis (passing through point O) of the ring. Calculate the magnitude and direction of $\underline{F}(z)$, the gravitational force exerted on mass m by the ring.



(40) (6) Given a thin solid plane quarter-circular disc of radius R and of area density σ kg m^{-2}. Calculate the moment of inertia I_{cm} about an axis which passes through the center of mass of the disc and perpendicular to the plane of the disc. Express your answer in terms of R and the mass M of the disc.

MATHEMATICAL FACTS

Elements of area: $dA = r\,dr\,d\theta$; $dA = dx\,dy$; $dA = 2\pi r\,dr$

Integrals: (a = const.)

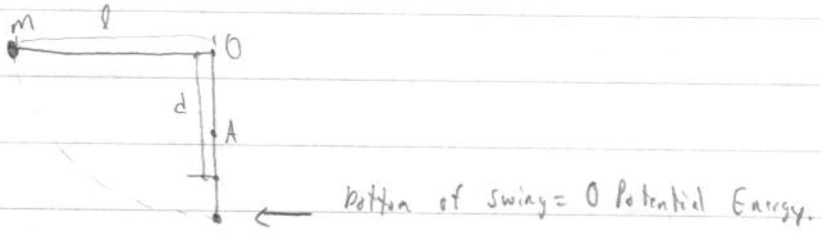
$$\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = \frac{-1}{(x^2+a^2)^{1/2}} + C$$

$$\int \frac{x\,dx}{(x^2+a^2)^{1/2}} = (x^2+a^2)^{1/2} + C$$

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{(x^2+a^2)^{1/2}} + C$$

$$\int \frac{dx}{(x^2+a^2)^{1/2}} = \ln\left(x + (x^2+a^2)^{1/2}\right) + C$$

1.



first find velocity of m when string is vertical.

Since the string is massless, we can use translational energy equations for m. We can use energy because there are no dissipative forces.

$PE_{top} \rightarrow KE$: $mgl = \frac{1}{2} m v_b^2$

$v_b = \sqrt{2gl}$ at bottom of string

Next, the string hits the peg. In order to complete the circular motion, m must have some velocity at top in order to provide the centripetal force. The minimum velocity needed corresponds to when only gravity provides the centripetal force: Tension is 0.

$mg = \frac{m v_{req}^2}{r}$ let $r = \frac{1}{2}$ distance from bottom of swing to A.

$v_{req} = \sqrt{rg}$

So the energy at the bottom goes to PE + KE:

$\frac{1}{2} m v_b^2 = mg(2r) + \frac{1}{2} m v_T^2$ v_T = velocity at top $= v_{req} = \sqrt{rg}$

$\frac{1}{2}(\sqrt{2gl})^2 = 2gr + \frac{1}{2}(\sqrt{rg})^2$

$gl = 2rg + \frac{1}{2} rg$

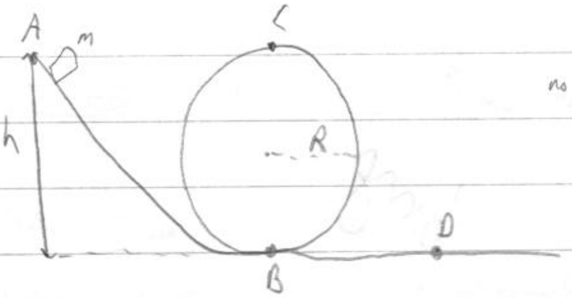
$l = \frac{5}{2} r$ So the radius (as measured) from the bottom of the string is $\frac{2}{5} l$

We want r in terms of l from the top:

$l - \frac{2}{5} l = \boxed{\frac{3}{5} l = d_c}$

30

2.



no friction

horizontal

a) we can use energy because there are no dissipative forces:

Initial $PE \rightarrow PE_C + \frac{1}{2} m V_{req}^2$

PE_C = Potential energy at C

V_{req} = velocity at C required to keep m in a circle

minimum velocity required!

$mgh = mg(2R) + \frac{1}{2} m V_{req}^2$

$gh = 2gR + \frac{1}{2}(\sqrt{Rg})^2$

$gh = 2gR + \frac{1}{2}Rg$

$\boxed{h = \frac{5}{2}R}$

$mg = \frac{mU^2}{R}$

$U = \sqrt{Rg}$

10/10

b) N_C = normal force at point C

forces at C:

N_C ↓mg

$N_C + mg = \frac{mV_T^2}{R}$

V_T = Velocity at top

$mgh = mg(2R) + \frac{1}{2} m V_T^2$

$2gh - 2gR = V_T^2$

$N_C = \frac{m(2gh - 2gR)}{R} - mg$

$= \frac{2mgh}{R} - 2mg - mg = \frac{2mgh}{R} - 3mg = N_C$

7/10

-1

4

B: N_B

mg

Here, no component of mg goes to help provide the normal force.

$N_B = \frac{mV_B^2}{R}$

V_B = Velocity at B

$mgh = mgR + \frac{1}{2} m v_B^2$

$2(gh - gR) = V_B^2$

-2

$= \frac{m(2gh - 2gR)}{R}$

$= \frac{2mgh}{R} - 2mg = N_B$

$V_B = \sqrt{2gh}$

$$N_B - N_C = \frac{2mgh}{R} - 2mg - \left[\frac{2mgh}{R} - 3mg\right]$$

$$= -2mg + 3mg = \boxed{\cancel{mg}}$$

6mg.

c. total energy is conserved from beginning to end (from A to D)

so $E_A = E_D$

$mgh = \frac{1}{2} m v_D^2$ $\qquad$ v_D = velocity at D

$\boxed{v_D = \sqrt{2gh}}$

5/5

d. $N_B - N_C$ does not depend on h or R; no matter what they are,

$\boxed{N_B - N_C}$ will still equal $\boxed{mg.}$

6mg?

5/5

$\oplus \leftarrow$

3. [1][2][3] $\overset{v}{\leftarrow}$ [4][5] Show 1+2 move off! m = mass of each block,

v_0 = velocity initial

29

elastic collision, so we can use energy

momentum conserved because of no net external force.

momentum initial = Momentum final

$2mv_0 = 2mv'$ $v_0 = v'$

$\frac{1}{2}(2m)v_0^2 = \frac{1}{2}(2m)v'^2$ So energy & momentum agree

$v_0^2 = v'^2$ $v_0 = v'$

if block 1 left with $2v'$! Mom.: $2mv_0 = m(2v')$

$v_0 = v'$

Energy: $\frac{1}{2}(2m)v_0^2 = \frac{1}{2}(m)(2v')^2$

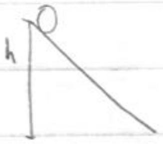
$mv_0^2 = 2mv'^2$ but from momentum we know $v_0 = v'$

So this would mean that there is more energy in the system after the collision than before, which violates conservation of energy. we cannot have more energy than we started out with. So therefore: $2m$ must move away with velocity v'.

Also show K.E is not conserv

-1

Any other combination of blocks coming off will not make energy & momentum agree! So $2m$ must come off with the same v', meaning blocks 1 + 2 must come off together.

4. a. h 

$I = cMR^2$ R = radius of Rigid body

c = some constant M = mass of Rigid body.

(for RI that varies with the body in question)

no dissipative forces, so use energy:

$Mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2$ $v = v_{cm} = R\omega$ NO SLIPPING

$Mgh = \frac{1}{2}(cM\cancel{R^2})\frac{v^2}{\cancel{R^2}} + \frac{1}{2}Mv^2$

$\cancel{M}gh = \frac{c}{2}\cancel{M}v^2 + \frac{1}{2}\cancel{M}v^2$

$gh = \left(\frac{c+1}{2}\right)v^2$

$\frac{2gh}{c+1} = v^2$

$v = \sqrt{\frac{2gh}{c+1}}$ ✓

So therefore v_{cm} at bottom is completely independent of M or R.

It only depends on the constant term of the rotational Inertia

b. $gh = \left(\frac{c+1}{2}\right)v^2$

$\boxed{\frac{2gh}{v^2} - 1 = c}$

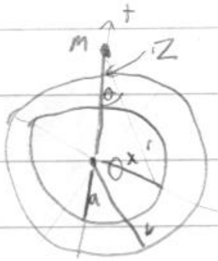
If we measure v at the bottom of the ramp and use the above equation, we can determine the coefficient c of rotational inertia for any body (rigid).

3/5

28/30

40/40

5.



σ = area mass density

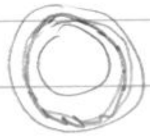
m = point mass

M = mass ring

r = distance from ring to m

$$F_G = \frac{GMm}{r^2}$$

$$dF = \frac{G\,dM\,m}{r^2} \qquad dM = 2\pi x\,dx\cdot\sigma$$! circular element of mass



By symmetry, all components of F_y cancel

So only component of F is $F_x = F\cos\theta$

So $$dF = \frac{G\cdot(2\pi x dx \sigma)\cdot m}{r^2}\cos\theta \qquad r^2 = x^2+z^2$$

$$dF = \frac{G(2\pi x dx \sigma)\cdot m}{x^2+z^2}\cos\theta \qquad \cos\theta = \frac{z}{\sqrt{z^2+x^2}} \qquad \left(\frac{z}{r}\right)$$

$$dF = \frac{2\pi G\sigma m\cdot x\,dx\cdot z}{(x^2+z^2)^{3/2}}$$

$$\int dF = F = \int_a^b \frac{2\pi G\sigma m z\cdot x\,dx}{(x^2+z^2)^{3/2}}$$

$$= 2\pi G\sigma m z\int_a^b \frac{x\,dx}{(x^2+z^2)^{3/2}} = 2\pi G\sigma m z\left[\frac{-1}{\sqrt{x^2+z^2}}\right]_a^b$$

$$= 2\pi G\sigma m z\left[\frac{-1}{\sqrt{x^2+b^2}} + \frac{1}{\sqrt{x^2+a^2}}\right]$$

direction is in $-\hat{z}$ direction (downward) as gravity is an attractive force

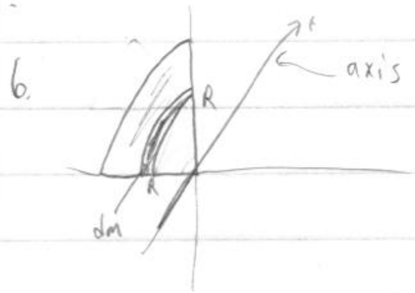
$\pi r\,dr = dm$ $\qquad M = \frac{\pi R^2}{2}$

$\int_0^R r^2 \cdot \pi r\,dr$

$= \pi \frac{R^4}{4} = \frac{1}{2} M R^2$

$\frac{40}{40}$

6.



σ = area density

uniform

$I = \int r^2\,dm \qquad dm = \frac{\pi}{2} r\,dr \cdot \sigma$

$I = \int_0^R r^2 \cdot \frac{\pi}{2} \cdot \sigma \cdot r\,dr = \frac{\pi\sigma}{2} \int_0^R r^3\,dr = \frac{\pi\sigma}{2} \cdot \frac{r^4}{4} \Big|_0^R = \frac{\pi\sigma}{2} \cdot \frac{R^4}{4} =$

$M = \frac{\pi R^2}{4} \cdot \sigma \qquad$ so $I = \frac{\pi\sigma}{4} \cdot R^2 \cdot \frac{R^2}{2} = \frac{1}{2} M R^2$ through axis at center of original disc

need to find center of mass of disc:

$x_{cm} = \frac{1}{M} \int_{-R}^{0} x\,dm \qquad dm = dx \cdot y \cdot \sigma = dx \sqrt{R^2 - x^2} \cdot \sigma, \quad M = \frac{\pi R^2 \sigma}{4}$

$= \frac{1}{M} \int_{-R}^{0} x \sqrt{R^2 - x^2}\,dx \cdot \sigma = \frac{\sigma}{M} \int_{-R}^{0} x \sqrt{R^2 - x^2}\,dx \qquad$ usub $= \left[\begin{array}{l} u = R^2 - x^2 \\ du = -2x\,dx \end{array} \right.$

$= \frac{\sigma}{M} \cdot -\frac{1}{2} \int \sqrt{u}\,du$

$= \frac{\sigma}{M} \cdot -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} = \frac{\sigma}{M} \cdot -\frac{1}{2} \cdot \frac{2}{3} (R^2 - x^2)^{3/2} \Big|_{-R}^{0} = \frac{-\sigma}{M} \cdot \frac{1}{3} \cdot R^3 + 0$

$= \frac{-\sigma \cdot 4}{\pi R^2 \sigma} \cdot \frac{1}{3} \cdot R^3 = -\frac{4R}{3\pi} = x_{cm}$

$y_{cm} = \frac{1}{M} \int_0^R y\,dy \qquad dm = dy \cdot x \cdot \sigma = dy \cdot \sqrt{R^2 - y^2} \cdot \sigma$

$= \frac{1}{M} \int_0^R y \sqrt{R^2 - y^2}\,dy =$ as done above $= \frac{\sigma}{M} \cdot -\frac{1}{2} \cdot \frac{2}{3} (R^2 - y^2) \Big|_0^R = 0 + \frac{\sigma}{M} \cdot \frac{1}{3} \cdot R^3$

$= \frac{4R}{3\pi} = y_{cm}$

So use parallel axis thm: $I = I_{cm} + Md^2$

$I - Md^2 = I_{cm} \qquad d = \left(\left(\frac{-4R}{3\pi}\right)^2 + \left(\frac{4R}{3\pi}\right)^2 \right) = \left(\frac{16R^2}{9\pi^2} \right) 2 = \frac{32R^2}{9\pi^2}$

$\frac{1}{2} M R^2 - \frac{M 32 R^2}{9\pi^2} = I_{cm}$