Midterm 2
EE40
Spring 2014

NAME: Solutions

Instructions
Read all of the instructions and all of the questions before beginning the exam.

There are 4 problems in this exam. The total score is 100 points. Points are given next to each problem to help you allocate time. Do not spend all your time on one problem.

IMPORTANT

- If you do not put your answers within the boxes labeled ‘Solution’ THEY WILL NOT BE COUNTED (no matter how correct they may be in the bottom left back corner of the third to last page of the exam.)

- If you have more than one solution in the box, that box will be given zero points.

Unless otherwise noted on a particular problem, you must show your work in the space provided, on the back of the exam pages or in the extra pages provided at the back of the exam.

Be sure to provide units where necessary.

GOOD LUCK!

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Problem 1 Warm-up (20 points)

Consider the circuit below.

\[ V_{\text{in}} \quad R \quad C \quad V_{\text{out}} \]

a) If $V_{\text{in}}$ is a DC source equal to 1 V, what is the steady-state value of $V_{\text{out}}$?

**Solution:**

\[ V_{\text{out}} = 0 \text{ V} \]

b) If instead, if $V_{\text{in}} = \begin{cases} 0 \text{ V for } t < 0 \\ 1 \text{ V for } t \geq 0 \end{cases}$ what is $V_{\text{out}}(t)$ for $t > 0$?

**Solution:**

\[ e^{-t/RC} \text{ Volts} \]

**Method 1:**

\[ V_{\text{out}}(t) = i(t) R = RC \frac{d}{dt} (1 - V_{\text{out}}) \]

\[ \therefore \quad \frac{dV_{\text{out}}}{V_{\text{out}}} = -\frac{dt}{RC} \Rightarrow V_{\text{out}}(t) = V_{\text{out}}(0) e^{-t/RC} \]

\[ V_{\text{out}}(t = 0) = 0 \Rightarrow V_{\text{out}}(t) = e^{-t/RC} \]

**Alternatively:**

\[ V_{\text{out}}(t) = V_{\text{out}}(t = 0) + \left[ V_{\text{out}}(t = 0) - V_{\text{out}}(t = 0) \right] e^{-t/RC} \]

\[ V_{\text{out}}(t) = e^{-t/RC} \text{ (V)} \]
Consider the circuit below.

\[ V_{in} \quad R \quad C \quad V_{out} \]

\[ V_{out}(t=\infty) = 0 \text{ V} \]
\[ V_{out}(t=0) = 1 \text{ V} \]

(c) If \( V_{in} \) is a DC source equal to 1 V, what is the steady-state value of \( V_{out} \)?

Solution:

\[ V_{out} = 1 \text{ V} \]

(d) If instead, if \( V_{in} = \begin{cases} 0 \text{ V for } t < 0 \\ 1 \text{ V for } t \geq 0 \end{cases} \) what is \( V_{out}(t) \) for \( t > 0 \)?

Solution:

\[ V_{out} = 1 - e^{-t/\tau} \text{ Volts.} \]
\[ V_{out}(t) = 1 - e^{-t/\tau} \]
Consider the circuit below.

\[ V_{in} \quad L \quad V_{out} \quad R \quad V_{out} \]

\( V_{in} \)

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**e)** If \( V_{in} \) is a DC source equal to 1 V, what is the steady-state value of \( V_{out} \)?

**Solution:**

\[ V_{out} = 1V \]

\[ V_{out}(t=0) = 0 \text{ V} \]

\[ V_{out}(t=\infty) = 1 \text{ V} \]

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**f)** If instead, if \( V_{in} = \begin{cases} \text{0 V for } t < 0 \\ \text{1 V for } t \geq 0 \end{cases} \) what is \( V_{out}(t) \) for \( t > 0 \)?

**Solution:**

\[ 1 - e^{-t/\tau} \text{ Volts} \]

\[ V_{out}(t) = V_{out}(t=\infty) + \left[ V_{out}(t=0) - V_{out}(t=\infty) \right] e^{-t/\tau} \]

\[ = 1 - e^{-t/\tau} \]

\[ \tau = L/R \]
Consider the circuit below.

![Circuit Diagram]

g) If $v_{in}$ is a DC source equal to 1 V, what is the steady-state value of $v_{out}$?

Solution:

$V_{out} = 0\, V$

$V_{out}(t = 0) = 1\, V$

$V_{out}(t = \infty) = 0\, V$

h) If instead, if $v_{in} = \begin{cases} 0 \, V & \text{for } t < 0 \\ 1 \, V & \text{for } t \geq 0 \end{cases}$ what is $v_{out}(t)$ for $t > 0$?

Solution:

$e^{-\frac{R}{L}t}$

$V_{out}(t) = 0 + (1 - 0) e^{-\frac{t}{\tau}}$; $\tau = \frac{L}{R}$.
Problem 2 (25 points)
Consider the circuit below; it is designed to measure \( C_{\text{meas}} \).

\[ V_{\text{in}}(t) \quad i_c(t) \quad V_{\text{out}}(t) \]

\( V_{\text{in}}(t) \) is 0 for \( t < 0 \) and follows the following graph for \( t \geq 0 \).

\[ V_{\text{in}} \quad 1V \]
\[ 1ms \]

a) Provide a plot of \( i_c(t) \) on the axes below.

\[
\begin{align*}
0 \leq t \leq 1ms & : \quad i_c(t) = C_{\text{meas}} \frac{dV_{\text{in}}}{dt} \\
1ms < t & : \quad \frac{dV_{\text{in}}}{dt} = 0 \quad \therefore i_c(t) = 0
\end{align*}
\]

* Otto West: “Apes don’t read philosophy.”
* Wanda: “Yes they do, Otto. They just don’t understand it. Now let me correct you on a couple of things, OK? Aristotle was not Belgian. The central message of Buddhism is not ‘Every man for himself.’ And the London Underground is not a political movement. Those are all mistakes, Otto. I looked them up.”

A Fish Called Wanda, 1988
b) Provide an expression for $v_{out}(t)$ for $0 < t \leq 1$ ms.

Solution:

\[ i_c(t) + C_f \frac{dv_{out}}{dt} = 0 \]

\[ C_f \frac{dv_{out}}{dt} = -C_{meas} \]

\[ v_{out}(t) = -C_{meas} \frac{t}{C_f} \]

\[ v_{out}(t) = -\left( \frac{1V_{ms}}{ms} \right) C_{meas} \frac{t}{C_f} \]

c) What is the value of $v_{out}(t)$ at 1 ms?

Solution:

\[ -\frac{C_{meas}}{C_f} \text{ Volts.} \]

\[ v_{out}(t=1\text{ms}) = -\frac{1V_{ms}}{ms} \times 1\text{ms} \times \frac{C_{meas}}{C_f} \]
Now consider the following circuit.

\[ \text{Solution: } V_{\text{out}}(t) = \frac{C_{\text{meas}}}{C_f} \left( 1 - e^{-\frac{t}{R C_{\text{meas}}}} \right) \text{ Volts} \]

\[ \text{Virtual ground. Assuming ideal op-amp.} \]

\[ V_B = e \]

\[ \frac{V_B}{R} = -C_f \frac{dV_{\text{out}}}{dt} \]

\[ V_{\text{out}} = -\frac{1}{RC_f} \int_0^t e^{-\frac{t'}{RC_{\text{meas}}}} dt' \]

\[ V_{\text{out}} = -\frac{RC_{\text{meas}}}{RC_f} (1 - e^{-\frac{t}{RC_{\text{meas}}}}) \]
Now consider the following circuit.

\[ V_{\text{in}}(t) \rightarrow \text{in} \rightarrow C_{\text{meas}} \rightarrow R \rightarrow \text{op-amp} \rightarrow C_{f} \rightarrow V_{\text{out}}(t) \]

\[ V_{\text{in}}(t) = \begin{cases} 0 \text{ V for } t < 0 \\ 1 \text{ V for } t \geq 0 \end{cases} \]

Provide an expression for \( V_{\text{out}}(t) \) for \( t > 0 \).

**Solution:** \[ -\frac{C_{\text{meas}}}{C_{f}} \left( 1 - e^{-t/(R C_{\text{meas}})} \right) \text{ Volts} \]

Since assuming ideal op-amp, \( C_{p} \) is shorted.

\[ V_{\text{out}}(t) = -\frac{C_{\text{meas}}}{C_{f}} \left( 1 - e^{-t/(R C_{\text{meas}})} \right) \]
Problem 3 (n points)
Consider the circuit below.

\[ R_L = 1 \text{ k}\Omega \quad R_C = 1 \text{ k}\Omega \quad R = 100 \Omega \]
\[ L = 1 \text{ mH} \quad C = 1 \text{ nF} \]

Switch 1 (S1) is closed (i.e. horizontal in the circuit above) for \( t < 0 \).
Switch 2 (S2) is open (i.e. raised and not connected) for \( t < 0 \).

S1 opens at \( t = 0 \).
S2 closes at \( t = 1 \times 10^{-6} \) s

a) Provide an expression for \( v_{ab}(t) \) for \( 0 < t < 1 \times 10^{-6} \) s. Note that \( v_{ab}(t) \) is the potential difference between node a and node b.

Solution:

\[ v_a = 0 \]

\[ v_b(t) = -\frac{R_L}{R} V_i e^{-\frac{t}{1\mu s}} \]

\[ i_L(t = 0) = \frac{V_i}{R} \]

\[ i_L(t = \infty) = 0 \quad -\frac{1}{R L} \]

\[ \therefore i_L(t) = \frac{V_i}{R} e^{-\frac{t}{1\mu s}} \]

\[ v_{ab}(t) = 10 V_i e^{-\frac{t}{1\mu s}} \]
Problem 3b) [15 pts total]

\[ A + \quad t = 1 \times 10^{-6} \text{ s} \]

\[ V_C (t = 1 \times 10^{-6}) = 0 \text{ V} \]

\[ V_C (t = \infty) = \frac{R_C}{R + R_C} \cdot V_1 \]

\[ V_C (t) = V_f + [V_0 - V_f] e^{-\frac{t-t_1}{\tau}} \]

\[ \tau = R_{eq} \cdot C = (R_C \parallel R) \cdot C \]

\[ V_{ab} (t) = V_C (t) - V_{RL} (t) \]

\[ = \left( \frac{R_C}{R + R_C} \right) V_1 \left( 1 - e^{-\frac{t-10^{-6}}{(R_C \parallel R) \cdot C}} \right) + \frac{R_L}{R} V_1 e^{-\frac{t}{10^{-6}}} \]

\[ = \frac{10}{11} V_1 \left( 1 - e^{-1.1 \times 10^{-7} (t - 10^{-6})} \right) + 10 V_1 e^{-\frac{t}{10^{-6}}} \]
Nigel Tufnel: "The numbers all go to eleven. Look, right across the board, 11, 11, 11 and..."
Marty DiBergi: "Oh, I see. And most amps go up to 10?"
Nigel Tufnel: "Exactly."

**Problem 4 (25 points)**
Consider the circuit below.

Provide an expression for $v_o$.

**Solution:**

$$32v_1 - 72\ v_2$$

Note that assuming ideal op-amps,

$$V_a = -4v_1, \ V_b = -8v_2$$

$$V_c = V_a = -4v_1; \ V_d = V_b = -8v_2$$

$$V_e = V_d = -8v_2$$

KCL @ $V_e$: \[ \frac{V_o}{4} - \frac{V_e}{4} = \frac{V_e}{5} - \frac{V_c}{5} \]

$$\frac{V_o}{4} = -2v_2 - 16v_2 + 8v_1$$

$$V_{out} = 32v_1 - 72v_2$$