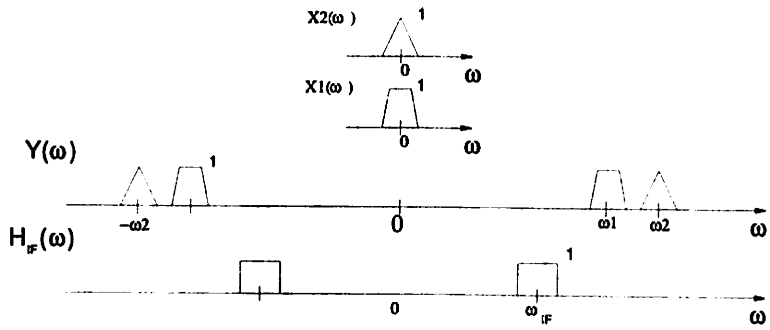
Key

Problem 1. Superheterodyne receiver (24 pts)

Two signals $x_1(t)$ and $x_2(t)$ have spectra (Fourier Transforms) $X_1(j\omega)$ and $X_2(j\omega)$ respectively, as shown in the figure below. You are given a signal $y(t)$ with spectrum $Y(j\omega)$, which contains 2 received signals, $x_1(t)\cos(\omega_1 t)$ (from station 1) and $x_2(t)\cos(\omega_2 t)$ (from station 2). A narrow bandpass filter $H_{IF}(j\omega)$ with fixed center frequency $\omega_{IF} < \omega_1$ is used to filter out interference.

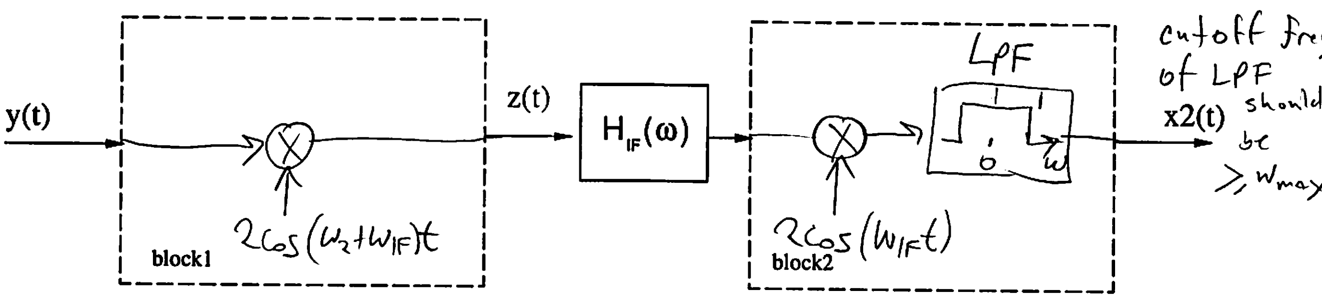


The block diagram below, when completed, should recover the original $x_2(t)$ from $y(t)$.

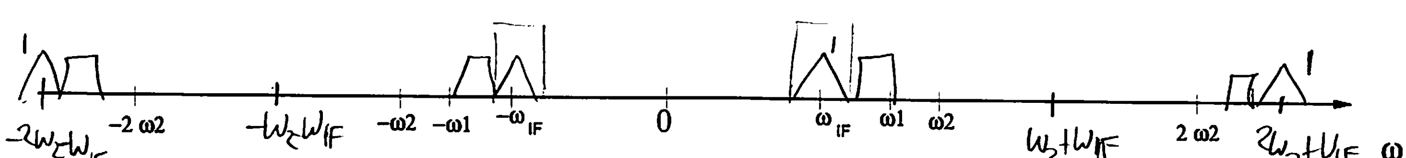
[7 pts] a. Add signal processing elements as necessary in *block1* such that the output $Z(j\omega)$ will have $X_2(j\omega)$ in the passband of the bandpass filter. For each element, specify amplitudes and frequencies as necessary.

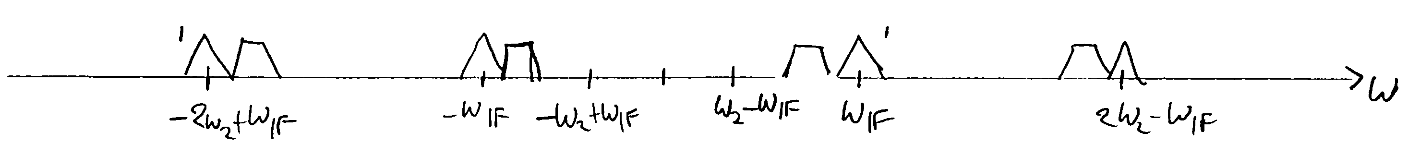
[10 pts] b. For your *block1* system, sketch the spectra $Z(j\omega)$, labelling key frequencies and amplitudes.

[7 pts] c. Add signal processing elements as necessary in *block2* such that the output of the block will match the original signal before transmission, $x_2(t)$. For each element, specify amplitudes and frequencies as necessary.



Z(jω)





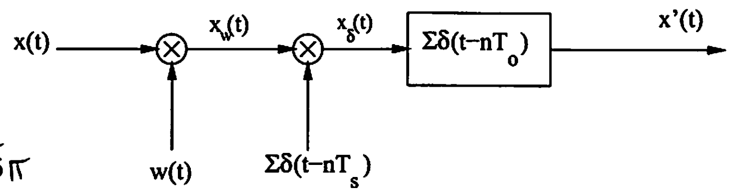
using $2\cos(\omega_2-\omega_{IF})$

Key.

Problem 2. Sampling and Discrete Fourier Transform (28 pts)

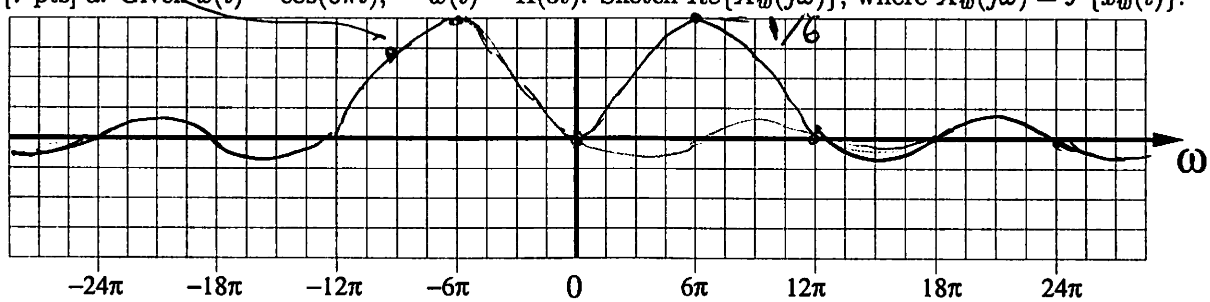
For parts a) and b), consider the system below, where $x(t) = \cos(6\pi t)$. Parts a) and b) may have different $w(t), T_s, T_o$ and should be answered independently. Sketch should label peak magnitude, and frequency of zero crossing(s) should match given scale.

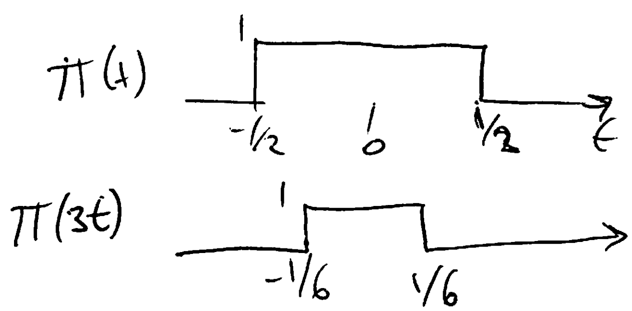
Note $\Pi(t) = u(t + 0.5) - u(t - 0.5)$.



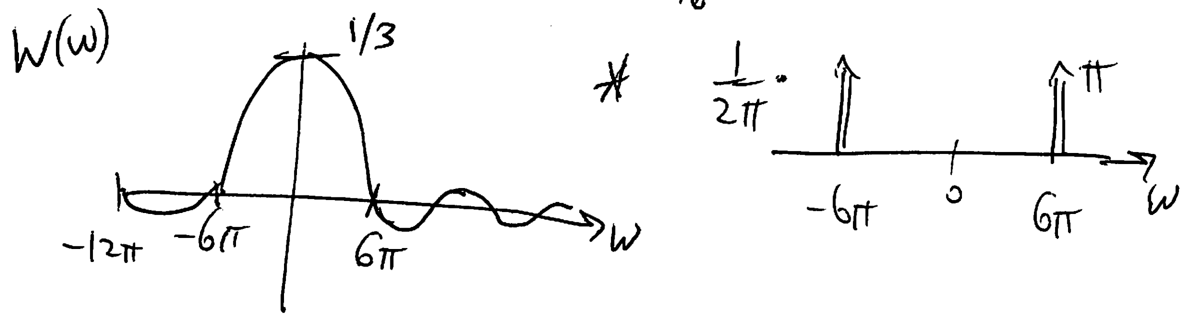
$\frac{2}{3\pi} + \frac{2}{15\pi}$

[7 pts] a. Given $x(t) = \cos(6\pi t)$, $w(t) = \Pi(3t)$. Sketch $Re\{X_w(j\omega)\}$, where $X_w(j\omega) = \mathcal{F}\{x_w(t)\}$:





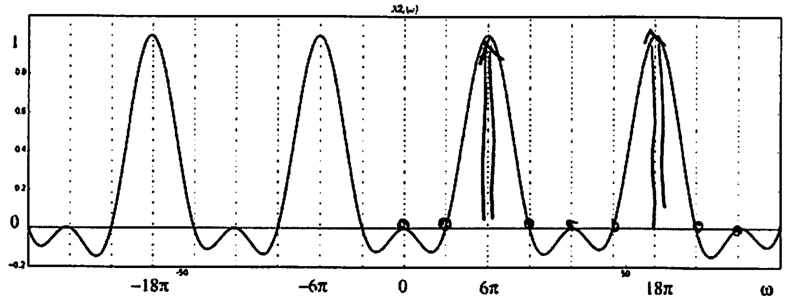
$$W(\omega) = \int_{-1/6}^{1/6} e^{-j\omega t}\,dt = \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-1/6}^{1/6} = \frac{2\sin \omega/6}{\omega}$$



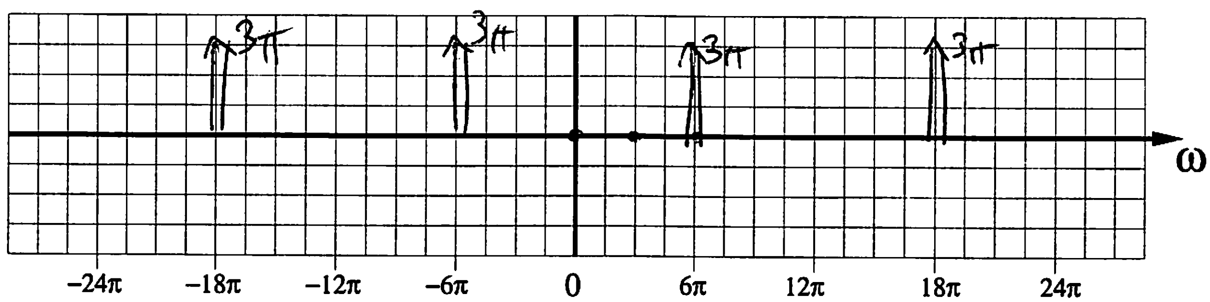
Key.

Problem 2. cont.

[7 pts] b. Given a windowed and sampled signal $x_{2\delta}(t)$ with spectrum $X_{2\delta}(j\omega)\}$:



Sketch $Re\{X_2'(j\omega)\}$ where $X_2'(j\omega) = \mathcal{F}\{x_2'(t)\}$, given $T_o = 2/3$ sec.



$$x_{2\delta}(t) \longrightarrow \boxed{\sum \delta(t - nT_0)} \longrightarrow x_2'(t)$$

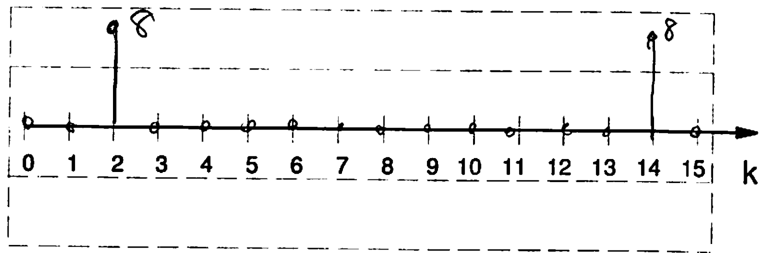
$$\sum \delta(t - nT_0) \overset{\mathcal{F}}{\longleftrightarrow} \frac{2\pi}{T_0} \sum \delta\left(\omega - \frac{k2\pi}{T_0}\right)$$

$$= 3\pi \sum \delta(\omega - 3k\pi)$$

Key

Problem 2. cont.

[6 pts] d. Given $x_3[n] = \cos(\pi n/4)$, sketch $X_3[k]$, the 16 point DFT of $x_3[n]$, labelling amplitudes.



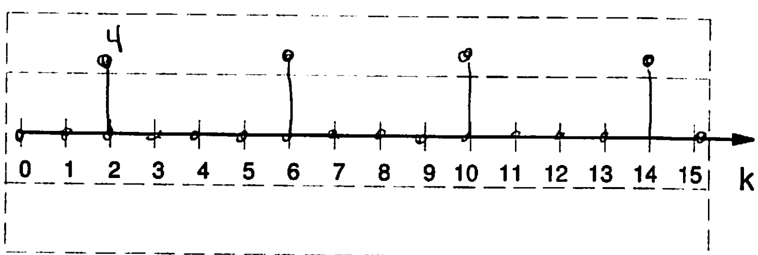
$$x_3[n] = \frac{1}{2}\left(e^{j\pi n/4} + e^{-j\pi n/4}\right)$$

$$X_3[k] = \frac{1}{2}\sum_{n=0}^{15}\left(e^{j\pi n/4} + e^{-j\pi n/4}\right)e^{-j2\pi nk/16} = \frac{1}{2}\sum_{n=0}^{15}\left(e^{j\frac{\pi n}{4}(1-k/2)} + e^{-j\pi n/4\,(1+k/2)}\right)$$

$1 - k/2 = 0 \Rightarrow k = 2$ $\qquad$ $1 + k/2 = 0 \Rightarrow k = -2$ (or $k = 14$)

$\frac{1}{2}\sum_{n=0}^{15} 1 = 8$ $\qquad$ all other terms 0.

[6 pts] e. Given $x_4[n] = x_3[n]$ for n even, and $x_4[n] = 0$ for n odd, sketch $X_4[k]$, the 16 point DFT of $x_4[n]$, labelling amplitudes.



$$X_4[k] = \sum_{n=0}^{15} x_4[n]e^{-j2\pi nk/16} = \left(x_4[0]e^{-j2\pi 0} + x[2]e^{-j2\pi 2k/16} + \cdots x[14]e^{-j2\pi 14k/16}\right)$$

$$= \sum_{m=0}^{7} x_3[2m]\,e^{-j2\pi 2mk/16} = \frac{1}{2}\sum_{m=0}^{7}\left(e^{j2\pi m/4} + e^{-j2\pi m/4}\right)e^{-j\pi mk/4}$$

$$= \frac{1}{2}\sum_{m=0}^{7}\left[e^{j\frac{\pi}{2}m(1-k/2)} + e^{-j\pi m/2\,(1+k/2)}\right]$$

$1 - \frac{k}{2} = 0, 4, 8$ $\qquad$ $1 + \frac{k}{2} = 0, 4, 8, 12$

$\frac{k}{2} = +1, -3, -7$ $\qquad$ $\frac{k}{2} = -1, 3, 7, 11$ $\qquad$ $k=2$: 4 $\qquad$ $k=-2$: 4.

$k = +2, -6, -14$ $\qquad$ $k = -2, 6, 14, 22$

Key

Problem 3. Laplace Transform (22 points)

A causal system with input $x(t)$ and output $y(t)$ is described by the differential equation:

$$\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 6y(t) = x(t).$$

$y \to Y(s)$

$\frac{dy}{dt} \to sY(s) - y(0^-)$

$\frac{d^2y}{dt^2} \to s^2Y(s) - sy(0^-) - \dot{y}(0^-)$

[11 pts] a. Find $Y(s)$ and $y(t)$ for $x(t) = 0$ (ZIR). $y(0^-) = 1$, $\frac{d}{dt}y(0^-) = 2$.

$Y(s) = \dfrac{s+7}{s^2+5s+6}$ $\qquad$ $y(t) = \left(-4e^{-3t} + 5e^{-2t}\right)u(t).$

$s^2Y(s) - sy(0^-) - \dot{y}(0^-)$
$+ 5sY(s) - 5y(0^-) + 6Y(s) = 0$
$(s^2 + 5s + 6)Y(s) = sy(0^-) + 5y(0^-) + \dot{y}(0^-)$
$= s + 5 + 2$

PFE $\dfrac{s+7}{(s+3)(s+2)} = \dfrac{A}{s+3} + \dfrac{B}{s+2}$

$A = -4 \quad B = 5$

[11 pts] b. Find $Y(s)$ and $y(t)$ for $x(t) = u(t)$ (ZSR). $y(0^-) = 0$, $\frac{d}{dt}y(0^-) = 0$.

$Y(s) = \dfrac{1}{s(s^2+5s+6)}$ $\qquad$ $y(t) = \left(\frac{1}{3}e^{-3t} - \frac{1}{2}e^{-2t} + \frac{1}{6}\right)u(t)$

$X(s) = \frac{1}{s}$

$(s^2+5s+6)Y(s) = \frac{1}{s}$

$Y($

PFE $\dfrac{1}{s(s+3)(s+2)} = \dfrac{A}{s+3} + \dfrac{B}{s+2} + \dfrac{C}{s}$

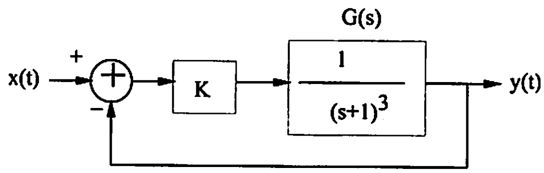
$A = 1/3 \quad B = -1/2 \quad C = 1/6$

$\frac{1}{3}(s)(s+2) + (-1/2)(s+3)s + \frac{1}{6}(s+3)(s+2)$

$= \frac{1}{3}s^2 + \frac{2s}{3} - \frac{1}{2}s^2 - \frac{3s}{2} + \frac{1}{6}s^2 + \frac{5s}{6} + 1$

$= 0 \cdot s^2 + s\left(\frac{4}{6} - \frac{9}{6} + \frac{5}{6}\right) + 1$

Problem 4. Feedback System (26 points)



$$\frac{\frac{K}{(s+1)^3}}{1+K/(s+1)^3} = \frac{K}{(s+1)^3+K}$$

[4 pts] a. Find the transfer function for the system above which has input $x(t)$ and output $y(t)$.

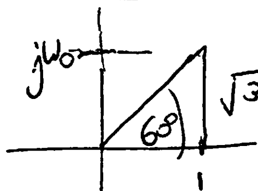
$$H(s) = \frac{Y(s)}{X(s)} = \underline{\frac{K}{(s+1)^3+K}} = \frac{K}{s^3+3s^2+3s+1+K}$$

[6 pts] b. Find the frequency ω_o at which the phase of $G(s)$ [$j\omega$] is $-180°$.

$$\omega_o = \underline{\sqrt{3}}$$

$$\angle G(j\omega) = -3\angle(1+j\omega) = -180°$$

$$\angle(1+j\omega) = 60°$$



[6 pts] c. For the frequency ω_o found above, what is the maximum K which could be used before the closed-loop system is unstable (this is the gain margin).

$$|j\omega_o+1|^3 = K$$

$$= \left(\sqrt{\omega_o^2+1}\right)^3 = 8$$

$$K < \underline{8}$$

$$1+\frac{K}{(j\omega_o+1)^3} = 0$$

$$\left|\frac{K}{(j\omega_o+1)^3}\right| = |-1|$$

$$\left|\frac{1}{(j\omega_o+1)^3}\right| \leq \frac{1}{K}$$

[6 pts] d. Find the sinusoidal steady state response of the closed-loop system with $K = 4$ to the input $x(t) = \cos(t)u(t)$, ignoring any transients. (Hint: phasors).

$$H(j\omega) = \frac{4}{(j\omega+1)^3+4}$$

$y(t) \approx \underline{4(\cos t + \sin t)}$ for large t.

$$H(j) = \frac{4}{j^3+3j^2+3j+5} = \frac{4}{-j+3j-3+5} = \frac{4}{2j+2} = \frac{2}{j+1}$$

$$x(t) = \left(\frac{e^{jt}+e^{-jt}}{2}\right)u(t),\ \omega = 1$$

$$j^3 = j \cdot j^2 = -j$$

$$y(t) \approx H(j)\frac{e^{jt}}{2} + H(-j)\frac{e^{-jt}}{2}$$

[4 pts] e. Without explicitly calculating, discuss the steady-state closed-loop response of the system to $x(t) = \cos(\omega_o t)u(t)$ using ω_o from part b and the K value from part c.

at $\omega_o = \sqrt{3}$ and $K = 8$, there is a pair of poles on the $j\omega$ axis $\pm j\sqrt{3}$. Thus the response will be unbounded to bounded input.