Mathematics 54.2
Exam 3, 1 May 2013
50 minutes, 50 points

NAME: ____________________ **ID:** ____________________

GSI: ____

INSTRUCTIONS:
You must justify your answers, except when instructed otherwise.
All the work for a question should be on the respective sheet.
This is a CLOSED BOOK examination, NO CALCULATORS are allowed.
NO CELL PHONE or EARPHONE use is permitted.
You ARE allowed to bring a notesheet, on one side of letter-size paper.
Please turn in your finished examination to your GSI before leaving the room.

Q1	
Q2	
Q3	
Q4	
Tot	
Ltr	

Question 1. (15 pts, 8+2+5)
(a) Draw a phase portrait of the following linear dynamical system:

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 1.4 & -.8 \\ 1.2 & -1.4 \end{bmatrix} \cdot \mathbf{x}$$

Explain your work, and indicate all significant features.
(b) The components x_1, x_2 represent populations, so are restricted to non-negative values. What best describes this system – cooperation, competition or a predator-prey model? Explain your thinking.
(c) Descibe the eventual outcome for the initial condition $x_1(0) = 1000, x_2(0) = 3500$, and for $x_1(0) = 1000, x_2(0) = 2750$

Question 2. (12 pts)
Find the Fourier *cosine* series of the function $f(x) = \sin x$ on the interval $[0, \pi]$.
Remark: You may find use for the trigonometric identities

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta, \quad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

Question 3. (8 pts)
Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

in the region $0 \le x \le \pi$, $t \ge 0$, with initial conditions $u(x,0) = \sin x(1 + \cos x \cos 2x \cos 4x)$ and boundary conditions $u(0,t) = u(\pi,t) = 0$ at all times $t \ge 0$.
Hint: You might find the formula $\sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha$ useful.

Question 4. (15 pts)
Describe Lagrange's method of solving an inhomogeneous ODE by 'variation of parameters', and illustrate it by finding the general solution to the equation

$$y''(t) - y(t) = \frac{2e^t}{1+e^t}.$$

Hint: The substitution $u = e^t$ should help with the integrals.

THIS PAGE IS FOR ROUGH WORK (not graded)