1. 40 points
Consider the matrix

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$

A has the value -2 as one of its eigenvalues. Find all the other eigenvalues. Find a non-zero vector v such that the limit of

$$A^n v$$

as n goes to (plus) infinity is the zero vector.

2. 40 points

Use the Gram-Schmidt process to find an orthonormal basis for $W = Span\,\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$, where:

$$\mathbf{u_1} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix}, \mathbf{u_2} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \mathbf{u_3} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

3. 20 points

Suppose A is a 3×3 matrix with real entries. Are the following statements true or false? Justify your answer.

3a.

If A has 3 linearly independent eigenvectors then it is diagonalizable.

3.b

If A is diagonalizable it must have 3 distinct eigenvalues.

3.c

If $A = PDP^{-1}$ then A has the same eigenvalues as D.

3.d

If A is diagonalizable then A is invertible.