Midterm 1 Solutions

Physics 7C, Fall 2013

October 1, 2013

1 Radio Antennas [30 pts]

(a) **[5 pts]** The wavelength is given by (from $c = \lambda f$),

$$\lambda = c/f = \frac{3 \times 10^8 \text{ m/s}}{100 \times 10^6 \text{ Hz}} = 3 \text{ m}.$$

(b) **[5 pts]** Depending on the mechanism considered, there are two possible answers:

(I) If we consider induced EMF ($\mathscr{E} = -d\Phi_B/dt$), we want as large Φ_B as possible, so the surface normal $\hat{n}$ should be parallel to the magnetic field $\vec{B}$ to maximize the magnetic flux, $\Phi_B = \vec{B} \cdot \vec{A} = \vec{B} \cdot (A\hat{n})$.

Since $\vec{E}$ and $\vec{B}$ are perpendicular to the propagation direction ($\hat{n}$), the surface normal needs to be perpendicular to the propagation direction in this case.

(II) Alternately (and this gives the actual maximum induced voltage), we can look at the voltage drop across the diameter of the loop caused by electric field. To maximize this, the surface normal should be perpendicular to $\vec{E}$; of the two possible directions, one of them is parallel to the propagation direction—in either way, we read the voltage difference across the diameter parallel to $\vec{E}$. And the voltage is,

$$V = Ed = E(2r).$$

(c) **[10 pts]** For both scenarios, we need to know E_{rms} at 100 m from the station. Intensity is given by,

$$I = P/A = \frac{P}{4\pi d^2} = \frac{400 \times 10^3 \text{ W}}{4\pi (100 \text{ m})^2} = (10/\pi) \text{ W/m}^2.$$

Intensity is related to E_{rms} by,

$$I = c\epsilon_0 E_{\text{rms}}^2,$$

so solving for the rms electric field magnitude,

$$\begin{aligned} E_{\text{rms}} &= \sqrt{\frac{I}{c\epsilon_0}} \\ &= \sqrt{\frac{10/\pi \text{ W/m}^2}{(3 \times 10^8 \text{ m/s}) \cdot (9 \times 10^{-12} \text{ J/m/(V}^2))}} \\ &= \sqrt{\frac{10^5 \text{ V}^2}{27\pi \text{ m}^2}} \approx \frac{\sqrt{10}}{9} 10^2 \text{ V/m} \approx 30 \text{ V/m}. \end{aligned}$$

The remaining steps depend on the mechanism considered:

(I) For the induced EMF, we need to compute,

$$|\mathscr{E}| = |A \; dB/dt|,$$

for the magnetic field,

$$B(t) = B_0 \cos(kx - \omega t),$$

and in the limit $\lambda \gg r$ (which it is, since $3 \text{ m} \gg 30 \text{ cm}$), we approximate magnetic field to be uniform over the cross-section, and the first time-derivative is,

$$|dB/dt| = \omega B_0 |\sin(kx - \omega t)|,$$

and the rms induced EMF is,

$$\mathscr{E}_{\text{rms}} = A\omega B_{\text{rms}},$$

and since the EM fields in EM wave are related by, $B = E/c$,

$$\begin{aligned} \mathscr{E}_{\text{rms}} &= A\omega E_{\text{rms}}/c = \pi r^2 (2\pi c/\lambda) E_{\text{rms}}/c \\ &= (2r E_{\text{rms}})(\pi^2 r/\lambda) \\ &= 2 \cdot (0.3 \text{ m}) \cdot (30 \text{ V/m}) \frac{\pi^2 \cdot (0.3 \text{ m})}{3 \text{ m}} \\ &\approx 20 \text{ V}. \end{aligned}$$

(II) For the voltage drop due to electric field, the distance along $\vec{E}$ is the diameter of the loop, $2r$, so,

$$V = E(2r) = 2 \cdot (0.3 \text{ m}) \cdot (30 \text{ V/m}) \approx 20 \text{ V}.$$

So it turns out for the given numbers, the two answers are within 20% of each other (but for smaller loops (or longer wavelengths), (II) gives larger voltage; for larger loops (or shorter wavelengths), approximation made for (I) may no longer hold).

(d) **[10 pts]** The distance from the original emitter to the receiver is $D_1 = 100$ m. The second emitter is 9 m away from the original receiver in the perpendicular direction, so the distance from the second emitter to the receiver is,

$$D_2 = \sqrt{(100 \text{ m})^2 + (9 \text{ m})^2}.$$

The path length difference is,

$$\Delta D = D_2 - D_1 = \sqrt{(100 \text{ m})^2 + (9 \text{ m})^2} - (100 \text{ m}),$$

and simplifying,

$$\Delta D = (100 \text{ m}) \left[\{1 + (9/100)^2\}^{1/2} - 1 \right].$$

We can use binomial expansion ($[1+\epsilon]^n \approx 1 + n\epsilon + \cdots$) to further simplify,

$$\Delta D \approx (100 \text{ m})[1 + (1/2) \cdot (0.0081) - 1] \approx 0.41 \text{ m}.$$

This is much less than $\lambda/2 = 1.5$ m (first destructive interference), so the two emitters are constructively interfering at the receiver's location, and the signal is stronger.

2 Thin Film of Oil [20 pts]

(a) **[5 pts]** For the air-to-oil reflection, there is a π phase shift, $\phi_1 = \pi$; for the oil-to-water reflection there is no phase shift on reflection, but the additional distance $2t$ traveled in oil causes phase shift of,

$$\phi_2 = 2\pi(2t/\lambda_{\text{oil}}) = 2\pi(2n_{\text{oil}}t/\lambda).$$

The total phase difference is,

$$\Delta\phi = \phi_2 - \phi_1 = 2\pi\left[2n_{\text{oil}}t/\lambda - 1/2\right],$$

which should equal $2\pi m$ (m integer) for constructive interference.

Solving for the thickness t,

$$t = \frac{(m+1/2)\lambda_R}{2n_{\text{oil}}} = \frac{\lambda_R}{4n_{\text{oil}}} = \frac{700 \text{ nm}}{6} \approx 100 \text{ nm},$$

picking $m = 0$ for minimum thickness.

(b) **[5 pts]** Using the same formula with the new wavelength,

$$t = \frac{\lambda_V}{4n_{\text{oil}}} = \frac{420 \text{ nm}}{6} = 70 \text{ nm}.$$

(c) **[5 pts]** Simplest way to find the minimum thickness that is constructively interfering for both λ_R and λ_V is to list the first few thicknesses for each wavelength, and see which ones coincide. The thicknesses are given by,

$$t = \frac{(m+1/2)\lambda}{2n_{\text{oil}}},$$

so the thicknesses for $m = 0, 1,$ and 2 for $\lambda = 700$ nm are,

$$t_R = 117 \text{ nm},\ 350 \text{ nm},\ 583 \text{ nm},$$

and the thickness for $m = 0, 1,$ and 2 for $\lambda = 420$ nm are,

$$t_V = 70 \text{ nm},\ 210 \text{ nm},\ 350 \text{ nm}.$$

So $t = 350$ nm is the thinnest oil film that is constructively interfering for both wavelengths.

(d) **[5 pts]** Given the thickness t, the wavelengths constructively interfering are given by,

$$\lambda = \frac{2n_{\text{oil}}t}{m+1/2}.$$

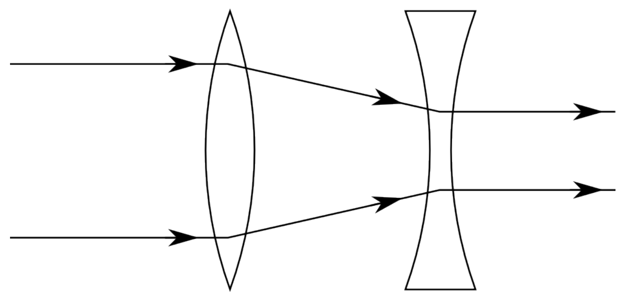
For $t = 350$ nm and $m = 0, 1, 2,$ and 3,

$$\lambda = 2100 \text{ nm},\ 700 \text{ nm},\ 420 \text{ nm},\ 300 \text{ nm}.$$

The two visible wavelengths 700 nm and 420 nm are the already known wavelengths; 2100 nm is infrared, and 300 nm is ultraviolet. With higher m, the wavelengths will get only shorter, so there are no other visible colors constructively interfering for this film.

3 Galilean Telescope [30 pts]

(a) **[5 points]** See below diagram (drawn for $l = f_o - f_e$, which forms virtual image at infinity; other distances will form a real or virtual image at a finite distance):



(b) **[10 points]** We work through the lens equations. For the objective lens,

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f_o},$$

solving for image distance d_i,

$$d_i = \frac{f_o d_o}{d_o - f_o},$$

and for $d_o \gg f_o$,

$$d_i \approx \frac{f_o d_o}{d_o} = f_o.$$

This image is real ($d_i > 0$) and inverted ($m = -d_i/d_o < 0$).

For the eyepiece, the object distance (signed distance between the image from the objective and the eyepiece) is given by,

$$d_o' = l - f_o$$

($d_o' < 0$ as expected for virtual object), and the lens equation reads,

$$\frac{1}{d_o'} + \frac{1}{d_i'} = \frac{1}{-f_e},$$

and solving for the final image distance,

$$d_i' = -\frac{f_e d_o'}{d_o' + f_e'} = -\frac{f_e(l - f_o)}{l - f_o + f_e} = \frac{f_e(f_o - l)}{l - (f_o - f_e)}.$$

For $l < (f_o - f_e)$, the image is virtual; for $f_o - f_e < l < f_o$, the image is real.

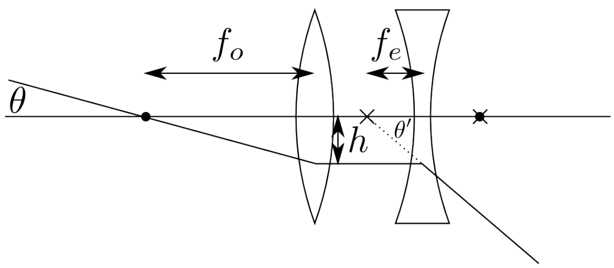
If final image is virtual, it is upright ($m = (-d_i/d_o)(-d_i'/d_o') < 0$); if the final image is real, it is inverted ($m = (-d_i/d_o)(-d_i'/d_o') > 0$).

(c) **[5 points]** For viewing with a relaxed eye, the final image should be a virtual image at infinity ($d_i' \to -\infty$). To obtain this, we let

$$l \to (f_o - f_e)$$

from below.

(d) **[10 points]** We derive the angular magnification for the geometry given by this sconfiguration. First, we start with the figure below, for an object of angular size θ.



The angle θ is given by,

$$\tan\theta = h/f_o,$$

and the angle θ' is given by,

$$\tan\theta' = h/f_e,$$

so solving both for h and equating the two expressions for h,

$$f_o \tan\theta = f_e \tan\theta',$$

and,

$$\frac{\tan\theta'}{\tan\theta} \approx \frac{\theta'}{\theta} = \frac{f_o}{f_e},$$

which is the angular magnification $M = \theta'/\theta$, under small-angle approximation on both angles.

4 Optics In Water [20 pts]

(a) **[5 pts]** From the lensmaker's equation,

$$\frac{1}{f} = \left(\frac{n_1}{n_2} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right),$$

plugging in the given numbers ($r_1 \to \infty$ and $r_2 = 40$ cm),

$$\frac{1}{f} = \left(\frac{1.6}{1} - 1\right)\left(0 + \frac{1}{40\text{ cm}}\right) = \frac{0.6}{40\text{ cm}} = \frac{3}{200\text{ cm}},$$

and $f \approx 67$ cm, with the positive sign indicating that it's a converging lens.

(b) **[5 pts]** Using the same lensmaker's equation with $n_2 = n_{\text{water}}$,

$$\frac{1}{f} = \left(\frac{1.6}{4/3} - 1\right)\left(0 + \frac{1}{40\text{ cm}}\right) = \frac{0.2}{40\text{ cm}} = \frac{1}{200\text{ cm}}.$$

So the lens is a converging lens of focal length 200 cm.

(c) **[5 pts]** For an air bubble of radius R, under the thin lens approximation (which is not valid for a bubble—but graded as correct for this problem),

$$\frac{1}{f} = \left(\frac{n_1}{n_2} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right),$$

with the radii of curvature R, $n_2 = 4/3$ and $n_1 = 1$,

$$\frac{1}{f} = \left(\frac{1}{4/3} - 1\right)\frac{2}{R} = -\frac{2}{4R},$$

so the bubble makes a diverging lens of focal length $f = -2R$, under thin lens approximation.

(Since the two curved surfaces are separated by $2R$, thin-lens approximation doesn't actually apply, and we need to use the thick-lens version of the lensmaker's equation (from Wikipedia, with appropriate changes for general application and differences in sign convention),

$$\frac{1}{f} = \left(\frac{n_1}{n_2} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2} - \frac{(n_1/n_2 - 1)d}{(n_1/n_2)r_1 r_2}\right),$$

where $r_1 = r_2 = R$, and $d = 2R$; so,

$$\frac{1}{f} = \left(-\frac{1}{4}\right)\left(\frac{2}{R} - \frac{(-1/4)(2R)}{(3/4)R^2}\right) = -\frac{1}{4}\left(\frac{2}{R} + \frac{2}{3R}\right).$$

Simplifying,

$$\frac{1}{f} = -\left(\frac{1}{2R} + \frac{1}{6R}\right) = -\frac{2}{3R}.$$

So the focal length is,

$$f = -3R/2,$$

which is still diverging.)

(d) **[5 pts]** The angle of incidence is $\theta = 45°$; in water, the critical angle is given by,

$$\sin\theta_c = \frac{n_{\text{water}}}{n_{\text{glass}}} = \frac{4/3}{8/5} = \frac{5}{6} \approx 0.83.$$

However, the angle of incidence gives,

$$\sin\theta = \sin(45°) = \frac{\sqrt{2}}{2} \approx 0.70,$$

and since $\sin\theta < \sin\theta_c$, $\theta < \theta_c$, and since the angle of incidence is smaller than the critical angle, total internal reflection does not happen in water.