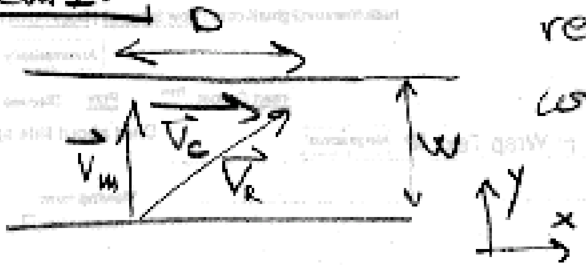
## Problem 1.

– Swimming straight across

relative speed of monkey with respect to shore

A




$$\vec{V_R} = \vec{V_M} + \vec{V_C}$$

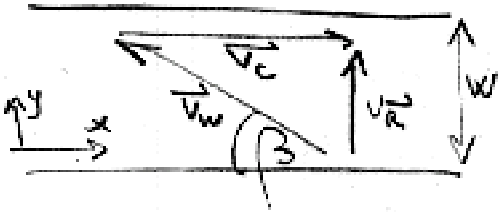
B

$x: \quad w = v_M t \longrightarrow \boxed{t = w / v_M}$

$y: \quad D = v_c t \Longrightarrow \boxed{D = w \frac{v_c}{v_M}}$

– Swimming upstream

C




Again $\vec{V_R} = \vec{V_M} + \vec{V_C}$ but now $\vec{V_R}$ points straight across the river.

$y: \quad V_R = V_M \sin\beta$

$x: \quad V_c = V_M \cos\beta \longrightarrow \boxed{\beta = \cos^{-1} V_c / V_M}$

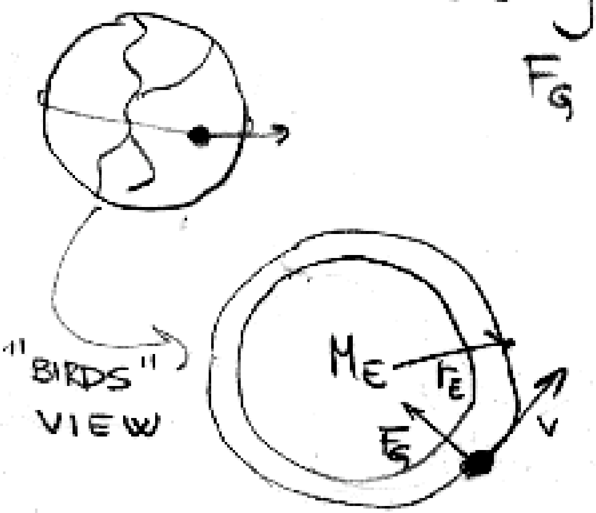
D

$y: \quad w = v_R t = v_M \sin\beta\, t = v_M \sqrt{1-\cos^2\beta}\; t$

$$t = w / v_M \sin\beta = \frac{w}{v_M \sqrt{1-\left(\frac{v_c}{v_M}\right)^2}} = \boxed{\frac{w}{\sqrt{v_M^2 - v_c^2}}}$$

## Problem 2.

The only force on the satellite:




$$F_G = G\frac{m M_E}{r_E^2} = mg = m a_{cp} = m\frac{v^2}{r_E}$$

$$\cancel{m} g = \cancel{m}\frac{v^2}{r_E} \qquad \boxed{v = \sqrt{g r_E}}$$

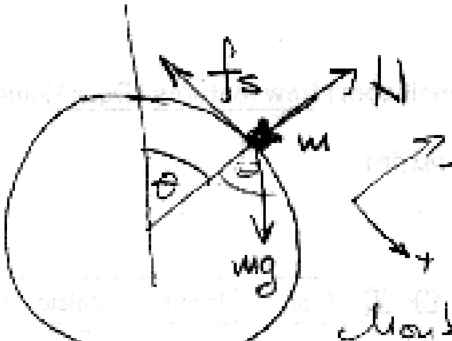
– on the surface:

$$F_G = G\frac{m M_E}{r_E^2} = mg = F_g$$

One period:

$$v = \frac{2\pi r_E}{T} = \sqrt{g r_E} \qquad T = \frac{2\pi r_E}{\sqrt{g r_E}} = \boxed{2\pi\sqrt{\frac{r_E}{g}}}$$

## Problem 3.1




$y$: $F_y^{net} = N - mg\cos\theta$

$x$: $F_x^{net} = mg\sin\theta - f_s$

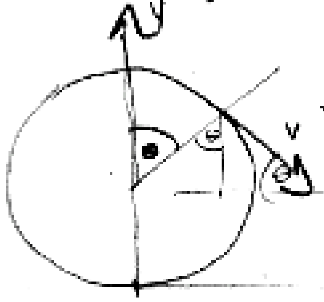
**A.** Monkey @ rest $\Rightarrow \vec{F}^{net} = 0 \Rightarrow$

$N = mg\cos\theta$ and $mg\sin\theta = f_s \le \mu_s N$

$mg = N/\cos\theta$ so $mg\sin\theta = \cancel{N}\tan\theta \le \mu_s \cancel{N}$

$\Rightarrow \tan\theta \le \mu_s$ and for max: $\boxed{\theta = \tan^{-1}\mu_s}$

**B.**




$v_{oy} = v\sin\theta$

$v_{ox} = v\cos\theta$

$a_y = -g$

$a_x = 0$

$y(t) = y_0 - v_{oy}t + \frac{1}{2}a_y t^2$

$= R(1+\cos\theta) - v\sin\theta t - \frac{1}{2}gt^2$

$x(t) = x_0 + v_{ox}t = R\sin\theta + v_0\cos\theta t$

when $y(t) = 0$, $x(t) = D = R\sin\theta + v_0\cos\theta t$

$0 = -R(1+\cos\theta) + v\sin\theta t + \frac{1}{2}gt^2$

$\rightarrow t = \dfrac{-v\sin\theta + \sqrt{v^2\sin^2\theta + 2Rg(1+\cos\theta)}}{g}$

and distance

$D = R\sin\theta + v_0\cos\theta t = \boxed{R\sin\theta + \frac{v\cos\theta}{g}\left(-v\sin\theta + \sqrt{v^2\sin^2\theta + 2Rg(1+\cos\theta)}\right)}$

where $\mu_s = \tan\theta$.

**C.** Monkey now accelerates upward with $a_y = a$:


$x$: $0 = F_x^{net} = -f_s\cos\theta_a + N\sin\theta_a$

$\Rightarrow \mu_s N \ge f_s = N\tan\theta_a$

$\mu_s \ge \tan\theta_a$ as before

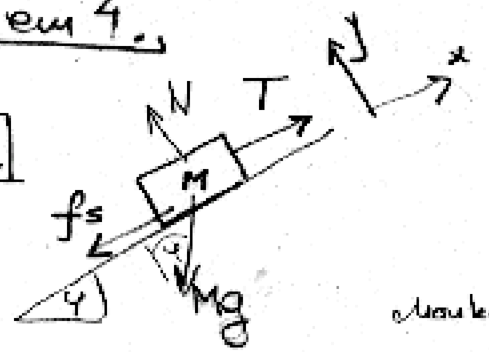
and $\boxed{\theta_a = \theta = \tan^{-1}\mu_s}$

**D.** The only difference from part B is $a_y = -(g+a)$

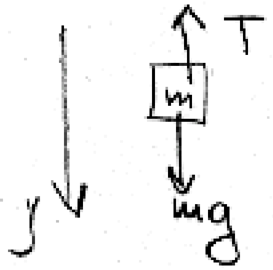
therefore $D_a = \boxed{R\sin\theta + \frac{v\cos\theta}{g+a}\left(-v\sin\theta + \sqrt{v^2\sin^2\theta + 2R(g+a)(1+\cos\theta)}\right)}$

Problem 4.

**A.**




System is @ rest and static friction on M is maxed out.




monkey: $mg - T = 0$

Box: $T - f_s - Mg\sin\varphi = 0$

$N - Mg\cos\varphi = 0$

$$f_s = T - Mg\sin\varphi = mg - Mg\sin\varphi$$

$$f_s \le \mu_s N = \mu_s Mg\cos\varphi \implies$$

$$\mu_s M\cancel{g}\cos\varphi \ge m\cancel{g} - M\cancel{g}\sin\varphi$$

and system starts moving up if

$$\boxed{\mu_s < \frac{m - M\sin\varphi}{M\cos\varphi}} \quad \text{and} \quad \boxed{m > M\sin\varphi}$$

**B.** monkey: $mg - T = ma$ same a

Box: $T - f_k - Mg\sin\varphi = Ma$

$N - Mg\cos\varphi = 0 \implies N = Mg\cos\varphi$

adding first two equations:

$$mg - f_k - Mg\sin\varphi = (M+m)a, \qquad f_k = \mu_k N$$

$$a = \frac{mg - \mu_k Mg\cos\varphi - Mg\sin\varphi}{M+m}$$

Kinematics $v_f^2 = v_i^2 + 2ah = 2ah$ $\qquad v_f = \sqrt{2ah}$

$$\boxed{v_f = \sqrt{\frac{2gh}{M+m}(m - \mu_k M\cos\varphi - M\sin\varphi)}}$$