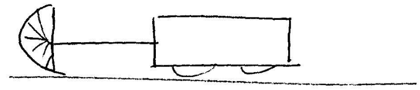
Section 1, Problem 1



$$x(t) = -At^2 + Bt - Ce^{-Dt}$$

a) $v(t) = \frac{dx}{dt} = -2At + B + CD e^{-Dt}$

(5) so $\boxed{v(0) = B + CD}$ Numerically $\boxed{v(0) = 110\ m/s}$

b) and $\boxed{v(1) = -2A + B + CDe^{-D}}$ Numerically $\boxed{v(1) = 83.67\ m/s}$

(3)

c) $\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{x(1) - x(0)}{1}$

(5) $\boxed{\bar{v} = -A + B + C(1 - e^{-D})}$ Numerically $\boxed{\bar{v} = 96{,}32\ m/s}$

d) $a(t) = -2A - CD^2 e^{-Dt}$

(5) so $\boxed{a(0) = -2A - CD^2}$ Numerically $\boxed{a(0) = -30\ m/s^2}$

e) and $\boxed{a(1) = -2A - CD^2 e^{-D}}$ Num. $\boxed{a(1) = -23.67\ m/s^2}$

(3)

f) $\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{v(1) - v(0)}{1}$

(4) $\boxed{\bar{a} = -2A + CD(e^{-D} - 1)}$ Num. $\boxed{\bar{a} = -26{,}33\ m/s^2}$

Question			
a)	Derivative (2)	Algebraic result (1)	Numerical (2)
b)		Algebraic (2)	Numerical (1)
c)	Definition (2)	Algebraic (2)	Numerical (1)
d)	Derivative (2)	Algebraic (1)	Numerical (2)
e)		Algebraic (2)	Numerical (1)
f)	Definition (1)	Algebraic (2)	Numerical (1)

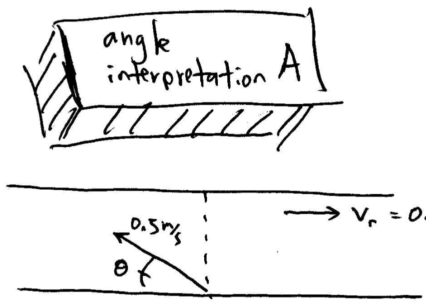
→ Rounded up or down on {neatness, comments, ...}

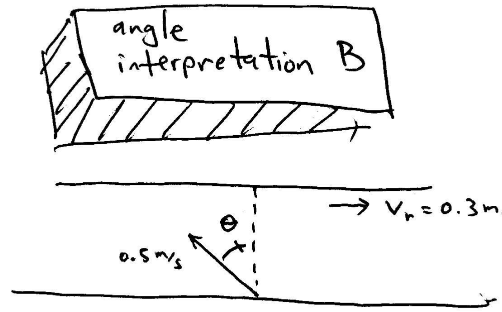
→ (-1) when no units.

If the calculus is correct without actual algebraic result they get all the points

Problem #2 10 pts

a.) 5 pts





Interpretation A:

$(0.5\,m/s)\cos\theta = 0.3\,m/s$

$\boxed{\theta = \cos^{-1}(3/5) = 53.1°}$

Interpretation B:

$(0.5\,m/s)\sin\theta = 0.3\,m/s$

$\boxed{\theta = \sin^{-1}(3/5) = 36.9°}$

b.) Interpretation A:

$\Delta t = \dfrac{\Delta y}{V_y} = \dfrac{40\,m}{(0.5)\sin(53.1°)}$

$\boxed{\Delta t = 100\,s}$

Interpretation B:

$\Delta t = \dfrac{\Delta y}{V_y} = \dfrac{40\,m}{(0.5)\cos(36.9°)}$

$\boxed{\Delta t = 100\text{ sec}}$

Grading

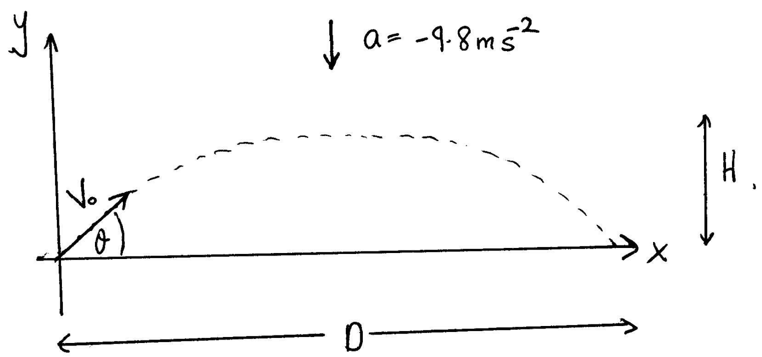
a.) -3 pts if incorrect trig function is used, or problem is set up incorrectly.

b.) -1 pt if velocity is not projected properly, but still use $\Delta t = \Delta x / v$

-1 pt for sign, algebra, or basic errors.

-2 pt if $\Delta t = \dfrac{\Delta x}{v}$ set up incorrectly.

Midterm 1.1 Problem 3 15pts



Given $D = 9.0\,m$ $\quad V_{ox} = 9.5\,ms^{-1}$

(a) $\text{time of flight} = \dfrac{D}{V_{ox}} = \dfrac{9.0\,m}{9.5\,ms^{-1}} = 0.95\,s.$

Since the motion is symmetric time of flight is twice time to max height

Thus $t_{max\,H} = \dfrac{0.95\,s}{2} = \boxed{0.475\,s}$

A nice diagram and something along this line of thought earned 4pts.

(b) Max Height $H = ?$

V_{oy} will be needed either in (b) or (c) depending on how you choose the problem. So let's find V_{oy}

$0 = V_{oy} - g\,t_{max\,height}$ $\quad$ Since $V_y = 0$ at H

$V_{oy} = (0.475)(9.8)\,ms^{-1} = 4.66\,m\,s^{-1}$

Finding V_{oy} correctly either numerically or just getting the expression earned 3pts.

b - Continued $y_{max} = ?$

$$y_{max} = y_0 + V_{0y}\, t_{max} - \frac{g}{2}\, t_{max}^2$$

$$H = 0 + (4.66)(0.475)\,m - (4.9)(0.475)^2\,m$$

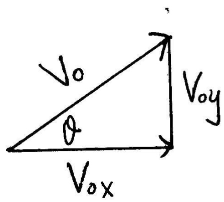
Thus

$$\boxed{H = 1.10\ m}$$

Using the correct expression for y_{max} and evaluating it correctly earned 4 pts.

(c) Find the angle θ :

look at the velocity vector diagram



$$\theta = \tan^{-1}\left(\frac{V_{0y}}{V_{0x}}\right)$$

$$\theta = \tan^{-1}\left(\frac{4.66}{9.5}\right)$$

Thus

$$\boxed{\theta \approx 26^\circ}$$

Using one of the trig. functions to correctly evaluate θ earned 4pts.

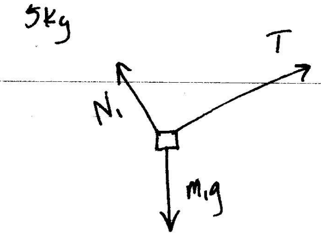
Total: 15 pts.

Note: Any valid method received full credit, it did not have to be done exactly this way, there are different but equivalent ways for calculating V_{0y}, H, and θ. In correct work received partial credit as long as it was relevent to problem and could have led to a correct solution.

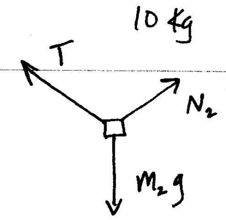
Solution

$\theta = 30°$

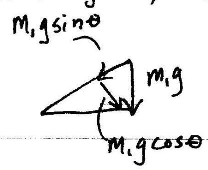
m_1:

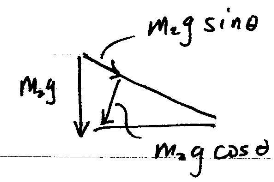


m_2:

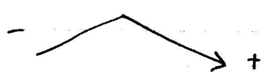


Resolve gravity vector:





Bent axis:



$\Sigma F = ma$:

$m_1 \rightarrow \quad T - m_1 g \sin\theta = m_1 a$

$m_2 \rightarrow \quad m_2 g \sin\theta - T = m_2 a$

b) $m_2 g \sin\theta - m_1 g \sin\theta - m_1 a = m_2 a$

$$a = \frac{(m_2 - m_1)}{m_1 + m_2} g \sin\theta = 1.63 \, m/s^2$$

c) $$T = m_1 g \sin\theta + m_1 a = m_1 g \sin\theta + m_1 \frac{(m_2 - m_1)}{m_1 + m_2} g \sin\theta$$

$$= \frac{2 m_1 m_2 g \sin\theta}{m_1 + m_2}$$

In terms of a: $T = m_1 (a + g \sin\theta)$

$$T = m_2 (g \sin\theta - a)$$

$$T = 32.7 \, N$$

Problem 4 Grading Scheme

FREE BODY DIAGRAMS: 5pts

Choice of axes, or sign of acceleration consistent with direction of motion - 7pts

Vector resolution - 4pts

Correct use of $\Sigma F = ma$ - 3 pts.

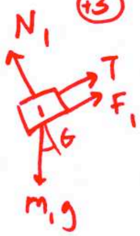
Algebra leading to "a" - 3pts

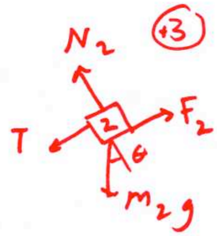
Algebra leading to "T" - 3pts.

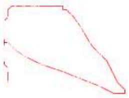
Alternately, you got 10 points for finding "a" using $\Sigma F = ma$ for the entire system.

5

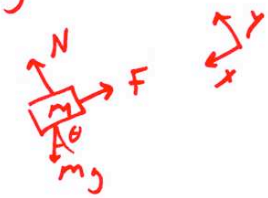
a) (+3) (+3)







b) For each block:



$\sum F_y = ma_y$

$\rightarrow N - mg\cos\theta = 0 \rightarrow N = mg\cos\theta$

$\sum F_x = ma_x$

$mg\sin\theta - f = mg\sin\theta - \mu N = mg\sin\theta - \mu mg\cos\theta = ma$

$\rightarrow a_1 = g(\sin\theta - \mu_1\cos\theta)$, $a_2 = g(\sin\theta - \mu_2\cos\theta)$

(+4) $\boxed{a_1 = 3.20\ m/s^2}$ $\boxed{a_2 = 2.35\ m/s^2}$ (+4)

with string, string is **taught** (+2) because $\boxed{a_1 > a_2}$ (+2) without string

c) y forces unchanged

$\sum F_{x_1} = m_1 g\sin\theta - \mu_1 m_1 g\cos\theta - T = m_1 a$ $(a_1 = a_2 = a)$

$\sum F_{x_2} = m_2 g\sin\theta - \mu_2 m_2 g\cos\theta + T = m_2 a$

$+ \quad (m_1 + m_2)g\sin\theta - g\cos\theta(m_1\mu_1 + m_2\mu_2) = (m_1+m_2)a$

$\rightarrow a = g\left(\sin\theta - \cos\theta \dfrac{\mu_1 m_1 + \mu_2 m_2}{m_1+m_2}\right) = \boxed{2.78\ m/s^2}$ (+4)

OR $F_{1net,\ no\ tension} = m_1 a_1$

Similarly, $F_{1net,\ with\ tension} = m_1 a$

But $F_{1\ net,\ with\ tension} = F_{1\ net,\ no\ tension} - T$

So $m_1 a = m_1 a_1 - T$

similarly, $m_2 a = m_2 a_2 + T$

So $\oplus$ $(m_1+m_2)a = m_1 a_1 + m_2 a_2 \rightarrow a = \dfrac{m_1 a_1 + a_2 m_2}{m_1 + m_2} = \boxed{\dfrac{a_1 + a_2}{2}}$!

↑ because $m_1 = m_2$

d) $T = m_1 g\sin\theta - m_1 a - \mu_1 m_1 g\cos\theta = \boxed{2.12\ N}$ (+3)

↑ we know all those quantities!