First midterm of Math 54, Fall 2012

Sep 13th 8:00-9:30am

Name ______________________

SID ______________________

Section ______________________

Note:

(1) *This is a closed book exam. No notes either. No calculators are allowed. All cellphones and other electronic devices should be off.*

(2) *Answers without explanation will not be accepted. Make sure to include your reasoning to get full credit. Write neatly and if you need extra space use the back of the pages containing the problems. If you do this make sure that the continuation is marked clearly.*

(3) *This exam contains 4 problems, each problem is worth the number of points indicated at the beginning of the problem.*

(4) *Read all questions carefully before you start trying to answer them.*

1	
2	
3	
4	
Total	

1. (20 points) The following statements are equivalent, except for one that is not. Identify it. Give an example showing that your choice is different from the other three.

   Let $\mathbf{A}$ be an $n \times n$ matrix.

   (a) The columns of $\mathbf{A}$ do not span $\mathbf{R^n}$.

   (b) The equation $\mathbf{Ax} = \mathbf{b}$ cannot be solved for all $\mathbf{b}$ in $\mathbf{R^n}$.

   (c) The linear transformation that sends an arbitrary $\mathbf{x}$ into $\mathbf{Ax}$ is one-to-one.

   (d) The reduced row-echelon form of $\mathbf{A}$ has one row consisting of all zeros.

2. (30 points) Given the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{pmatrix}$$

find all solutions to $\mathbf{Ax}=\mathbf{b}$ for $\mathbf{b}=[3, 9, 13]$.

3. (25 points)

Given the three vectors $\mathbf{v_1} = (1, -3, 2)$, $\mathbf{v_2} = (2, -6, h)$ and $\mathbf{v_3} = (5, -7, 0)$ in $\mathbf{R}^3$

a) For what values of h is $\mathbf{v_3}$ in the span of the other two vectors?

b) For what values of h is the set of three vectors a linearly independent set?

4. (25 points)

Label the following statements true or false. In each case either give an explanation or a counterexample. Read all questions carefully.

i. Subsets of linearly dependent sets of vectors in $\mathbf{R}^n$ are linearly dependent

ii. Subsets of linearly independent sets of vectors in $\mathbf{R}^n$ are linearly independent

iii. Any homogeneous system of linear equations has at least one solution