University of California, Berkeley

FINAL EXAMINATION, Spring 2012
DURATION: 3 hours

Department of Mathematics

MATH H53 Honors Multivariable Calculus

Examiner: Sean Fitzpatrick

Total: 100 points

Family Name: ______________________________
(Please Print)

Given Name(s): ______________________________
(Please Print)

Please sign here: ______________________________

Student ID Number: ______________________________

No aids, electronic or otherwise, are permitted, with the exception of the formula sheet provided with your exam. Partial credit will be given for partially correct work. Please read through the entire exam before beginning, and take note of how many points each question is worth.

Good Luck!

FOR GRADER'S USE ONLY	
Problem 1:	/7
Problem 2:	/12
Problem 3:	/10
Problem 4:	/12
Problem 5:	/10
Problem 6:	/15
Problem 7:	/10
Problem 8:	/14
Problem 9:	/10
TOTAL:	/100

1. Suppose you need to know an equation of the tangent plane to a surface S at the point
[7] $(2, 1, 3)$. You don't have an equation for the surface S, but you know that the curves

$$\vec{r}_1(t) = \langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \rangle$$
$$\vec{r}_2(u) = \langle 1 + u^2, 2u^3 - 1, 2u + 1 \rangle$$

both lie in S. Find an equation of the tangent plane to S at $(2, 1, 3)$.

2. Consider the function $f(x, y) = 4xy - 2x^4 - y^2$.

[6] (a) Find and classify the critical points of f.

[6] (b) Find the absolute maximum and minimum of f subject to the constraint $2x-y=1$, if they exist. If either the maximum or the minimum does not exist, explain why.

3. Let T be the region in $\mathbb{R}^3$ bounded by the surfaces $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{2 - x^2 - y^2}$. Sketch the region, then compute its volume using whichever coordinate system seems best to you.

[10]

4. Let D be the region in the first quadrant of the xy-plane bounded by the curves $y = x^2$, $y = 2x^2$, $x = y^2$ and $x = 4y^2$.

[3] (a) Sketch the region D.

[9] (b) Evaluate the integral

$$\iint_D \left(\frac{y^2}{x^4} + \frac{x^2}{y^4} \right) dx\, dy.$$

5. Let $\vec{F} = (2xz + y^2)\hat{\imath} + 2xy\hat{\jmath} + (x^2 + z^3)\hat{k}$.

[5] (a) Show that $\vec{F}$ is conservative, and find a potential function for $\vec{F}$.

[5] (b) Compute $\displaystyle\int_C \vec{F} \cdot d\vec{r}$, where C is given by $\vec{r}(t) = \langle t^2, t+1, 2t-1 \rangle$, with $t \in [0,1]$.

6. Let $\vec{F}$ be a vector field with continuously differentiable components.

[5] (a) Let S_1 and S_2 be two surfaces with a common boundary $C = \partial S_1 = \partial S_2$. Explain, with sketches, how S_1 and S_2 must be oriented in order to ensure that

$$\iint_{S_1} (\nabla \times \vec{F}) \cdot d\vec{S} = \iint_{S_2} (\nabla \times \vec{F}) \cdot d\vec{S}.$$

[3] (b) Explain why we should expect that $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S} = 0$ if S is a closed surface.

(c) Evaluate $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$ if S is the hemisphere given by $x^2 + y^2 + z^2 = 1$, $z \geq 0$,
[7] and $\vec{F} = y\hat{\imath} - x\hat{\jmath} + zx^3y^2\hat{k}$.

[10] 7. Verify the Divergence Theorem for $\vec{F}(x,y,z) = ||\vec{r}||\vec{r}$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, and S is the spherical surface $x^2 + y^2 + z^2 = 9$.

8. Recall (or look up on your formula sheet) the definition of differentiability discussed at length in class.

[5] (a) Prove that $f(x, y) = x^2 + y^2$ is differentiable for all $(x, y) \in \mathbb{R}^2$.

[9] (b) Discuss the meaning of the definition of the differentiability. In particular, how is the derivative related to the original function? How would you describe it geometrically? Explain what you think is most significant, and why.

9. A hypocycloid is the curve traced out by a marked point P on a circle of radius b as it rolls without slipping along the interior of a second circle with center O and radius $a > b$.

[5] (a) Determine parametric equations for the hypocycloid.
(Hint: Let $O = (0, 0)$, and let the initial position of P be $(a, 0)$. There should be an obvious choice of parameter.)

[5] (b) Find the length of the hypocycloid (for one trip around the big circle) in the case $a = 4$ and $b = 1$.