Mathematics 54.2
Midterm 1, 20 February 2013
50 minutes, 50 points

NAME: ______________________ **ID:** ______________________

GSI: ____

INSTRUCTIONS:
You must justify your answers, except when told otherwise.
All the work for a question should be on the respective sheet.
This is a CLOSED BOOK examination, NO NOTES and NO CALCULATORS are allowed.
NO CELL PHONE or EARPHONE use is permitted.
Please turn in your finished examination to your GSI before leaving the room.

Q1	
Q2	
Q3	
Tot	
Ltr	

1. TRUE-FALSE Questions (24 points) ∀∃

Circle the correct answer. No justification needed.

Correct answers carry 1.5 points, wrong ones carry 1.5 points penalty.

You may leave any question blank.

You will not get a negative total on any group of eight questions.

T	F	Two 5×7 matrices with the same column space have the same reduced row echelon form.
T	F	If the linear transformation $L : \mathbf{R}^2 \to \mathbf{R}^2$ takes $[1, 2]^T$ to $[2, 3]^T$ and $[3, 4]^T$ to $[5, 6]^T$, then L takes $[2, 3]^T$ to $[3, 4]^T$.
T	F	The columns of a matrix A are linearly independent if and only if the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.
T	F	If A is an invertible matrix, then the linear map $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one and onto.
T	F	There exists only one reduced row echelon matrix of size 4×3 and rank 3.
T	F	The dimensions of the row space and of the column space of a matrix A agree, whether or not A is square.
T	F	The pivot columns of any matrix A form a basis for $\mathrm{Col}(A)$.
T	F	If the column space of a 5×6 matrix is not $\mathbf{R}^5$, then the nullspace is at least 2-dimensional.

T	F	Every plane in $\mathbf{R}^3$ is a linear subspace.
T	F	The map $T : \mathbf{R}^2 \to \mathbf{R}^2$ which rotates vectors about the origin through an angle $-\pi/6$ is a linear transformation.
T	F	Two matrices with the same row space also have the same nullspace.
T	F	The columns of a matrix A are linearly independent if the system $A\mathbf{x} = \mathbf{0}$ has $\mathbf{x} = \mathbf{0}$ as a solution.
T	F	A linearly independent collection of vectors in $\mathbf{R}^{13}$ contains no more than 13 vectors.
T	F	If the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_4 \in \mathbf{R}^7$ are linearly independent, then in row-reducing the matrix whose columns are the $\mathbf{v}_i$ you will find exactly four rows of zeroes.
T	F	If two matrices A, B have the same reduced row echelon form, then their row spaces agree.
T	F	For any two 2×2 matrices A, B, we have $(AB)^T = A^T B^T$.

Question 2. (15 pts, 12+1+2)
(a) For the matrix A below, find bases for the row space, column space, nullspace and left nullspace. Describe your procedure clearly.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 6 & 7 & 10 \\ 2 & 4 & 5 & 7 \end{bmatrix}$$

(b) Briefly describe the relation between the left nullspace and the column space of a matrix.
(c) For what values of h is the vector $[h, h+1, h]^T$ in $\mathrm{Col}(A)$? Explain.

Question 3. (11 pts, 7+4)

(a) For each of the following matrices, find the inverse, or explain why it is not invertible:

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -3 \\ 3 & -4 \end{bmatrix}; \quad B = \begin{bmatrix} 1 & -2 & 5 \\ 2 & -3 & 4 \\ 3 & -4 & 4 \end{bmatrix}$$

(b) The linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ satisfies

$$T \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \mathbf{e}_1, \quad T \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = -\mathbf{e}_2, \quad T \begin{bmatrix} 5 \\ 4 \\ 4 \end{bmatrix} = \mathbf{e}_3.$$

Determine the standard matrix of T.

(Here, $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are the standard basis vectors $[1,0,0]^T, [0,1,0]^T, [0,0,1]^T$ of $\mathbf{R}^3$.)

THIS PAGE IS FOR ROUGH WORK (not graded)