Ö. Savaş NAME: **SOLUTIONS**

8:10-9:00, Friday, February 25, 2011

ME 106 FLUID MECHANICS

EXAM 1 – open class notes and bspace notes, no external communication

1. (20+5=25%) The Pitot-static tube of an A380 jumbo jet flying at 13 km altitude, its stated service ceiling, in standard atmosphere is reading a pressure differential of 1.0×10^4 Pa. You may consult the table in your class notes for atmospheric data.

(a) Determine the speed of the aircraft.

(b) Determine the Mach number of the aircraft.

At 13 km, $T = 216.65$K, $p_\infty = 1.65 \cdot 10^4$ Pa, and $\rho_\infty = 0.265$kg/m^3.

$$p_0 = \Delta p + p_\infty = 2.65 \cdot 10^4 Pa \quad \rho_o = \rho_\infty (p_0/p_\infty)^{1/\gamma} = 0.371 \; kg/m^3$$

$$\frac{\gamma}{\gamma-1}\frac{p_\infty}{\rho_\infty} + \frac{1}{2}u_\infty^2 = \frac{\gamma}{\gamma-1}\frac{p_0}{\rho_0} \quad \Longrightarrow \frac{1.4}{1.4-1}\frac{1.65\cdot 10^4}{0.265} + \frac{1}{2}u_\infty^2 = \frac{1.4}{1.4-1}\frac{2.65\cdot 10^4}{0.371} \quad \Longrightarrow u_\infty = 253 \, m/s$$

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{1.4 \times 287 \times 216.65} = 295 m/s \quad \Longrightarrow M_\infty = u_\infty / a_\infty = 253/295 = 0.86$$

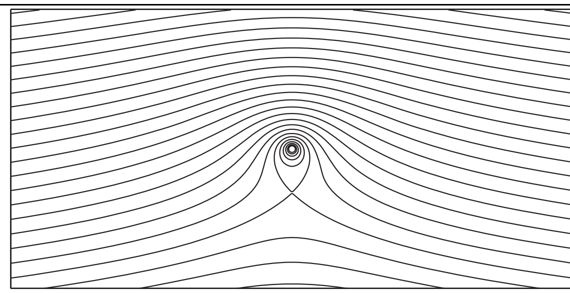
2.(20+5=25%) Construct and sketch the streamline equation for the flow field given in cylindrical coordinates

$$\mathbf{u} = (u_r, u_\theta) = \left(\cos\theta, \frac{1}{r} - \sin\theta\right)$$

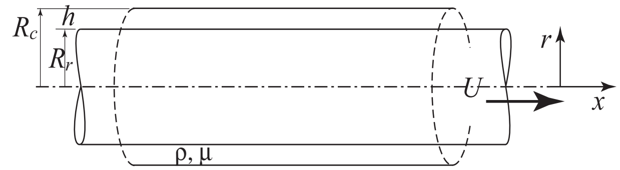
Hint: remember the chain rule; $d(fg) = g\ df + f\ dg$

$$\frac{dr}{u_r} = \frac{rd\theta}{u_\theta} \Longrightarrow \frac{dr}{\cos\theta} = \frac{rd\theta}{\frac{1}{r} - \sin\theta} \Longrightarrow \frac{dr}{r} - \sin\theta\, dr - r\ \cos\theta\ d\theta \Longrightarrow d(\ln r) - d(r\sin\theta) = 0 \Longrightarrow \ln r - r\sin\theta = cons.$$

Point vortex in uniform flow.



3. (25%) Determined the force per unit length required to pull at velocity U a rod of radius R_r out of a cylinder of inner radius R_c filled with an oil of viscosity μ. The rod and the cylinder are concentric. The gap between them $h = (R_c - R_r)$ is much smaller than their diameters, $h/R_c < h/R_r \ll 1$ so that you may wish to use planar flow approximation. Of course, you are always welcome to use the exact solution developed in class, but at your own peril!



Approximate Solution:

$$F/L = 2\pi R_r \times \tau = 2\pi R_r \times \mu \frac{U}{h} = 2\pi R_r \mu \frac{U}{R_c - R_r} = \frac{2\pi\mu U}{R_c/R_r - 1}$$

Exact Solution:

From class notes we have, in the absence of pressure gradient,

$$u = A \ln r + B$$

which, along with the no-slip boundary conditions, yields $u(R_r) = U$ and $u(R_c) = 0$.

$$\frac{u(r)}{U} = \frac{\ln(r/R_c)}{\ln(R_r/R_c)}$$

The shear stress on the surface of the rod is

$$\tau = \mu \frac{du}{dr}\bigg|_{R_r} = \mu A/R_r = \frac{\mu U}{R_c \ln(R_r/R_c)}$$

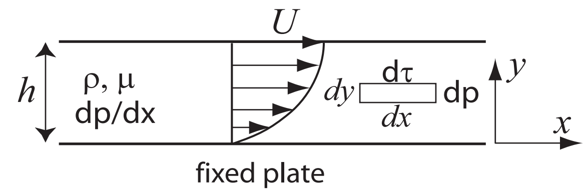
hence, the force to move it, per unit length, is

$$F/L = 2\pi R_r \times \tau = 2\pi R_r \times \frac{\mu U}{R_c \ln(R_r/R_c)} = \frac{2\pi\mu U}{(R_c/R_r)\ln(R_r/R_c)} \approx \frac{2\pi\mu U}{(R_c/R_r)\left[1 - (R_r/R_c)\right]} = \frac{2\pi\mu U}{R_c/R_r - 1}$$

where we made use of Taylor's expansion of $\ln(1+x) \approx x$ when $x \ll 1$.

4.(10+10+5=25%)

Consider the flow fluid of viscosity μ between two parallel infinite plates which are h apart. The top plate is moving to the right at constant velocity of U in its plane and the bottom plate is fixed. There is a constant negative pressure gradient of $dp/dx < 0$ acting on the fluid in the gap. Determine the value of $P = dp/dx$ for which the shear stress on the top plate vanishes.



Hint: Write the force balance for an infinitesimal fluid strip and apply the boundary conditions as you go along when integrating.

Boundary conditions: $u(0) = 0$, $u(h) = U$, required $\tau = \mu\, du/dy(h) = 0$.
Using the infinitesimal strip, we write force balance

$$dp\ dy = d\tau\, dx \Longrightarrow \frac{dp}{dx} = \frac{d\tau}{dy} = P = constant$$

By integrating, we deduce

$$\tau(y) = Py + A = \mu\frac{du}{dy} \Longrightarrow A = -hP$$

where we made us of the vanishing shear condition at $y = h$. We now write

$$\frac{du}{dy} = \frac{P}{\mu}(y - h) \Longrightarrow u(y) = \frac{P}{2\mu}(y^2 - 2hy)$$

here, we used the no slip condition at $y = 0$. Finally, we deduce

$$P = -\frac{2\mu U}{h^2}$$