Phys 7C Mid Term I Solutions

YU section

Q1

(a)

Condition for standing wave: $\lambda = 2L, L, \frac{2L}{3}$ etc.

$\therefore f = \frac{c}{\lambda} = \frac{c}{2L}, \frac{c}{L}, \frac{c}{2L/3} \ldots$

$= \frac{1}{2}\left(\frac{c}{L}\right), \frac{2c}{2L}, \frac{3}{2}\left(\frac{c}{L}\right), \ldots$

$\therefore$ separation between the frequencies

$= \frac{c}{2L} = \frac{3\times10^{10}\ \text{cm/s}}{2\times 12\ \text{cm}} = \frac{1}{8}\times10^{10}\ \text{Hz}$

$= \boxed{1.25\times10^{9}\ \text{Hz}}$

(b) intensity of the laser $= \frac{1\ \text{mW}}{0.2\ \text{cm}^2} = \frac{10^{-3}\ \text{W}}{0.2\times(10^{-2})^2\ \text{m}^2}$

$= \frac{10^{-3}}{0.2\times10^{-4}}\ \frac{\text{W}}{\text{m}^2} = 5\times10\ \frac{\text{W}}{\text{m}^2} = \underline{50\ \frac{\text{W}}{\text{m}^2}}$

$= uc$, u = energy density

$\therefore u = \frac{50}{3\times10^{8}\ \text{m/s}}\ \frac{\text{W}}{\text{m}^2} = \frac{50}{3\times10^{8}}\ \frac{\text{Joule}}{\text{m}^3}$

$= \underline{\frac{5}{3}\times10^{-7}\ \frac{\text{J}}{\text{m}^3}}$

$u = \epsilon_0 \langle E^2 \rangle$. If $E = E_0 \sin(kx-\omega t)$

$\langle E^2 \rangle = \frac{E_0^2}{2}$, E_0 amplitude of electric field

$\therefore u = \epsilon_0 \frac{E_0^2}{2}$

$E_0 = \sqrt{\frac{2u}{\epsilon_0}} = \sqrt{\frac{2\times 5\times10^{-7}}{3\times 8.85\times10^{-12}}}$ MKS unit or V/m

$= \sqrt{\frac{10^{-6}}{26.55\times10^{-12}}} = \frac{\sqrt{10^{6}}}{5.15}\ \text{V/m} \approx 1.9\times10^{2}\ \frac{\text{V}}{\text{m}}$

$= \underline{194\ \text{V/m}}$

$S = uc$

$= \epsilon_0 \langle E^2 \rangle c$

$= \frac{\epsilon_0 E_0^2 c}{2}$

Similarly $u = \frac{1}{\mu_0}\langle B \rangle^2 = \frac{B_0^2}{2\mu_0}$

$\therefore B_0 = \sqrt{\frac{2\times5\times10^{-7}}{3}\times(4\pi\times10^{-7})}\ \text{T}$

~~= [illegible]~~

$= \sqrt{\frac{10}{3}\times4\pi\times10^{-14}}\ \text{T}$

$= \boxed{6.47\times10^{-7}\ \text{T}}$

Q. 2

(a) $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

u = object distance = 70 cm

f = focal length of mirror = $\frac{R}{2}$ = 50 cm ($f > 0$ since mirror is focusing)

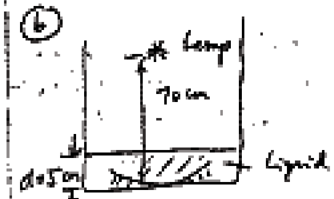
v = image distance from mirror

$\frac{1}{v} = \left(\frac{1}{50} - \frac{1}{70}\right)\frac{1}{cm} = \frac{0.0057}{cm}$

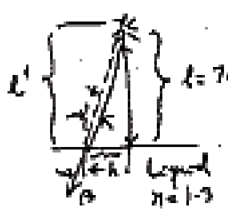
$\boxed{v = 175 \text{ cm}}$

Image is real since $v > 0$

(b)



Refraction at air / liquid interface will cause the lamp to appear at a different distance from mirror.



Snell's law:

$\sin\alpha = n \sin\beta$

For small $\alpha + \beta$

$\frac{\alpha}{\beta} \simeq n = 1.3$

Let l = distance of lamp from liquid = 65 cm.

l' = apparent distance of lamp from liquid due to refraction

$\frac{\alpha}{\beta} = \frac{h/l}{h/l'} = \frac{l'}{l}$ $\therefore$ $l' = \left(\frac{\alpha}{\beta}\right)l = nl$

$= 1.3 \times 65$ cm

$= \underline{84.5 \text{ cm}}$

Total distance of lamp from mirror after refraction = $l' + 5$ cm = $\underline{89.5 \text{ cm}}$

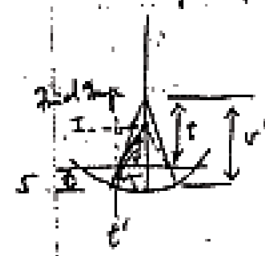
For this new object distance the image of lamp will now appear at v' where

$\frac{1}{89.5} + \frac{1}{v'} = \frac{1}{f} = \frac{1}{50}$

or $\frac{1}{v'} = \left(\frac{1}{50} - \frac{1}{89.5}\right)\frac{1}{cm}$

$\therefore$ $\underline{v' = 113.3 \text{ cm}}$

The reflected rays have to emerge from the liquid back into air. Again refraction at the liquid surface will make the image appear closer.



Based on same argument to derive the ratio between $l + l'$ we find that

$\frac{t}{t'} = \frac{l'}{l} = n$

$\therefore$ $t' = \frac{t}{n} = \frac{(v'-5)\text{ cm}}{n} = \frac{108.3}{n}$ cm

$= 83.3$ cm

$\therefore$ distance of image from mirror = $(83.3 + 5)$ cm

$= \boxed{88.3 \text{ cm}}$

Image is real again

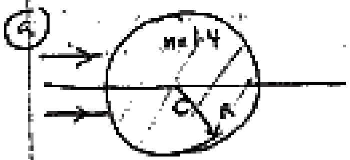
$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}$ $\frac{1}{\infty} + \frac{1.4}{d_i} = \frac{0.4}{1}$)

$d_i = 3.5$ cm

$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}$ $\frac{1.4}{-1.5} + \frac{1}{d_i} = \frac{1-1.4}{-1}$

Q. 3

10 (a)



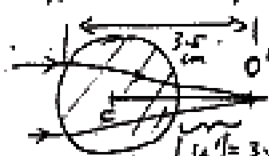
C = centre
R = radius = 1
n = 1.4 > 1
Assume parallel rays are incident on lens from left
u = object distance = ∞

$\frac{1}{u} + \frac{n}{v} = \frac{|n-1|}{R}$

$\therefore \frac{1}{\infty} + \frac{n}{v} = \frac{n-1}{R}$

$\therefore v = \left(\frac{n}{n-1}\right) R = \frac{1.4}{0.4} \, 1 \text{ cm} = 3.5 \text{ cm}$

This image formed by refraction at the 1st spherical surface will form a virtual object O' for the second spherical surface.



Let u' = distance of O' from 2nd spherical surface = −1.5 cm since it is virtual

$u' = 3.5 - 2 = 1.5$ cm

2nd radius is −ve > 0

$$\frac{n}{-1.5} + \frac{1}{v'} = \frac{n-1}{R}$$

$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}$

where v' = image of O' after refraction
(this image = focus of spherical lens)

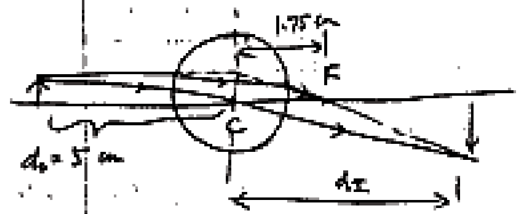
$\therefore \frac{1}{v'} = \frac{1.4-1}{R=0} + \frac{1.4}{1.5} \frac{1}{\text{cm}} = 0.4 + 0.933 \frac{1}{\text{cm}}$ $\frac{1}{d_o} + \frac{1.4}{d_i} = \frac{1.4-1}{R}$

or $v' = 0.75$ cm

The position of the image from center C = (1 + 0.75) cm
= **1.75 cm**

$\frac{1}{-1.5} + \frac{1.4}{d_i} = \frac{0.4}{-1}$

15 (b) The solution of this problem is simplified if we assume that the focal length of the lens is at F which is at a distance of 1.75 cm from C



The ray paths are equivalent to that of a converging lens located at C with f = 1.75 cm.

$\therefore \frac{1}{d_o} + \frac{1}{d_I} = \frac{1}{1.75}$

or $\frac{1}{d_I} = \left(\frac{1}{1.75} - \frac{1}{5}\right)\frac{1}{\text{cm}} = \frac{0.37}{\text{cm}}$

or **$d_I = 2.67$ cm**

This image is real since $d_I > 0$ &
linear magnification = $m = \frac{d_I}{d_o} = \frac{2.67}{5} =$ **0.54**

5 (c) Lens Maker Equation $\frac{1}{f} = \frac{|n-1|}{n_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

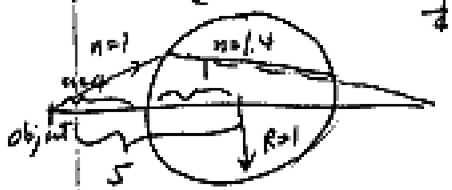
In air f = 5 ∴ $\frac{1}{5} = (1.5-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

If lens is now in a medium with $n_2 = 1.3$
then its focal length = f'

$\frac{1}{f'} = \frac{|n - n_2|}{n_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \left(\frac{0.2}{1.3}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$\therefore \frac{f'}{f} = \frac{0.5}{0.2} \cdot 1.3 = 2.5 \times 1.3$ or **f' = 16.25 cm**

alternate solution to
Quest. 3(b)

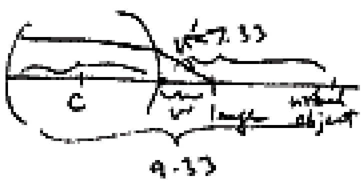


$\frac{1}{5}+\frac{1.4}{v}=\frac{(1.4-1)}{1}$

$\frac{1.4}{v} = 0.4-\frac{1}{5}$

$\frac{1.4}{v} = 0.4-0.25=0.15$

$v = \frac{1.4}{0.15} = 9.33$



$-\frac{1.4}{7.33}+\frac{1}{v'}=\frac{0.4}{1}$

$\frac{1}{v'} = 0.4+\frac{1.4}{7.33}$

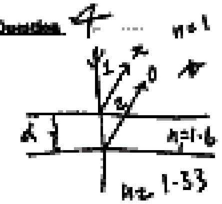
$= 0.4+0.19=0.59$

$v' = 1.69$

distance from C = $1.69+1 = \underline{2.69\,cm}$

Question 4

(a)



path difference between beam 1 & 2 = 2d

phase difference

$= 2d\left(\frac{2\pi}{\lambda}\right)n_1 - \pi$

Constructive interference

$\Rightarrow \frac{4\pi nd}{\lambda} - \pi = 0, 2\pi, 4\pi \ldots$

or $\frac{4\pi nd}{\lambda} = \pi$ $\quad \lambda = 4nd = 3200\,nm$

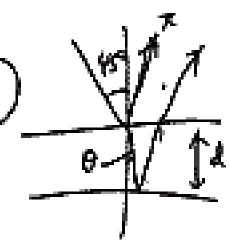
$3\pi \quad \lambda_1 = \frac{4nd}{3} = 1066.7$

$5\pi \quad \lambda_2 = \frac{4nd}{5} = 640$

$7\pi \quad \lambda_3 = \frac{4nd}{7} = 457$

$9\pi \quad \lambda_4 = \frac{4nd}{9} = 356$

(b)



path difference = $\frac{2d}{\cos\theta}$

$\sin 45° = 1.6\sin\theta$

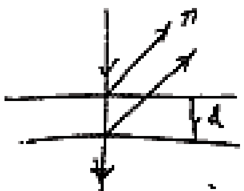
$\therefore \theta = 26.2°$

$\therefore$ path diff $= \frac{2d}{\cos 26.2°} = 1.115 \times 2d$

$\therefore \frac{4nd}{\lambda \cos 26.2°} = 1.115 \times 3200\,nm = 3577\,nm$

$\lambda_2 \Rightarrow 713\,nm$, $\lambda_3 \Rightarrow 509.6$, $\lambda_4 =$ ~~396 nm~~

Question 4 (c)



The transmitted light will show maximum intensity if the ~~transmitted~~ reflected light is minimum i.e. the reflected light destructively interfere

This ⇒ path difference $= \frac{4\pi n_1 d}{\lambda} - \pi = \pi, 3\pi, 5\pi, \ldots$

or $\frac{4\pi n_1 d}{\lambda} = 2\pi, 4\pi, 6\pi, \ldots$

$\frac{n_1 d}{\lambda} = \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$

or $\lambda = 2n_1 d, n_1 d, \frac{2n_1 d}{3}, \frac{n_1 d}{2};$

$n_1 d = 1.6 \times 500\text{ nm} = 800\text{ nm}$

$\therefore \lambda = 1600, ~~800~~, 533, 400, \ldots$