Math 54. **First Midterm**

1. (8 points) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 7 \\ 1 & 1 & 2 \end{bmatrix}$, if it exists. Use the algorithm introduced in Chapter 2.

2. (8 points) A matrix A and an echelon form of A are given here:

$$A = \begin{bmatrix} 1 & 2 & -1 & 1 & -1 \\ -2 & -4 & 3 & -3 & 0 \\ 1 & 2 & -3 & 3 & 3 \\ 1 & 2 & -2 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a). Write the solution set of the homogeneous system $A\vec{x} = \vec{0}$ in parametric vector form (i.e., as a linear combination of fixed vectors, in which the weights are allowed to take on arbitrary values).

(b). Give a basis of $\operatorname{Nul} A$.

(c). Give a basis of $\operatorname{Col} A$.

3. (5 points) Let A be an $m \times n$ matrix and let $\vec{b}$ be a vector in $\mathbb{R}^m$. Let $A' = \begin{bmatrix} A & \vec{b} \end{bmatrix}$ be the augmented matrix. Explain carefully why the equation $A\vec{x} = \vec{b}$ is consistent if and only if $\operatorname{rank} A = \operatorname{rank} A'$.

4. (8 points) Let $\vec{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 3 \\ -1 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 4 \\ 1 \\ 6 \\ -2 \end{bmatrix}$. Let $H = \text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

(a). Find a subset of $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ that is a basis for H. Explain how you know it is a basis for H.

(b). Let $\mathcal{B}$ be the basis you found in part (a), and let $\vec{x} = \vec{v}_1 + \vec{v}_2 + \vec{v}_3$. Find the $\mathcal{B}$-coordinate vector $[\vec{x}]_{\mathcal{B}}$ of $\vec{x}$.

5. (6 points) Let A be an $m \times n$ matrix, and let $\vec{b}$ and $\vec{c}$ be vectors in $\mathbb{R}^m$. Assume that both equations $A\vec{x} = \vec{b}$ and $A\vec{x} = \vec{c}$ are consistent. Show that the equation $A\vec{x} = \vec{b} + 7\vec{c}$ is consistent.

6. (8 points) Use Cramer's Rule to solve for x_2 in the linear system

$$\begin{aligned} 2x_1 \quad\quad + 3x_3 &= 2 \\ 3x_1 \quad\quad + 5x_3 &= 3 \\ 8x_1 + x_2 \quad\quad\quad &= 0 \end{aligned}$$

7. (7 points) Let $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation given by

$$T(x_1, x_2, x_3) = (9x_1 + 4x_2 + 7x_3, 2x_1 + 3x_3) \ .$$

(a). Find the standard matrix for T.

(b). Find a basis for the range of T. Explain your reasoning.