Physics 7A, Section 1 (Chiao)
Final Exam

University of California at Berkeley

Monday, May 16, 2005, 12:30-3:30 pm, 1 and 4 Leconte

IMPORTANT: Print your name, student ID number, GSI name, and your discussion section number on the front of your blue book.

This exam contains 5 questions, and will be graded out of a total of 100 points. You should answer all the questions to the best of your ability. You are allowed both sides of three sheets of **handwritten** notes, and the use of a calculator, but no QWERTY keyboards are allowed. Express all numerical results to 3 significant figures. The following are useful constants: The acceleration due to Earth's gravity in the Bay Area is $g = 9.80$ m/s^2. The Earths's radius is $r_E = 6.38 \times 10^3$ km. The mass of the Earth is $M_E = 5.97 \times 10^{24}$ kg. Newton's constant for universal gravitation is $G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}.

Please show all your work in your blue book. Explain the steps in your reasoning in coherent, English sentences. Define all symbols that you use. If you do not show relevant work for any part of the problem, you will not be awarded any credit, even if the answer is correct. If you recognize that an answer does not make physical sense, and you do not have time to find your error, write that you know that the answer cannot be correct, and explain how you know this to be true. (We will award some credit for recognizing there is an error.) For full credit, explain your reasoning carefully, show all steps neatly, and **box** your answers. Cross out any work you decide is incorrect, with an explanation in the margin.

Do the easiest problems first. You may answer the questions in any order you wish, but please clearly label each problem by number to ensure that it is properly graded.

DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO. STOP ALL WORK WHEN TOLD TO SO SO AT THE END OF THE EXAM. GOOD LUCK!!!

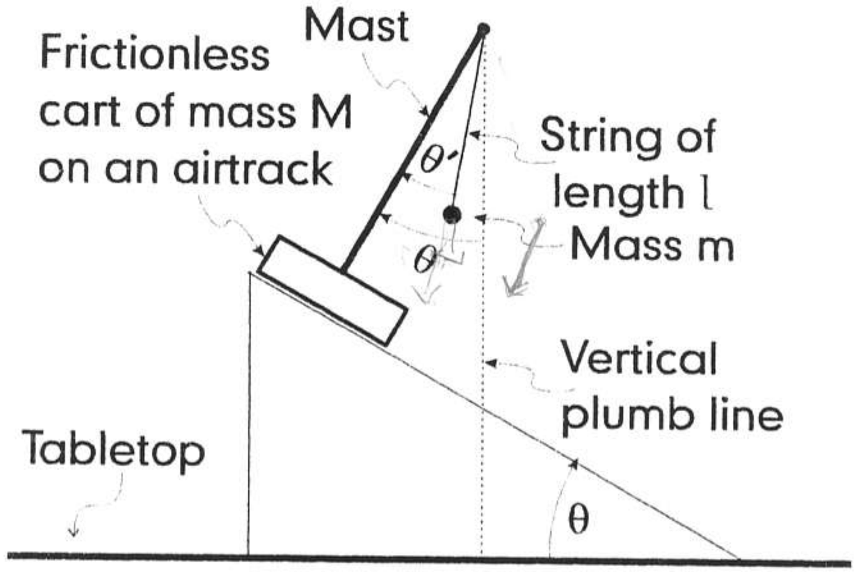

Figure 1: Figure for Problem 1.

PROBLEM 1: A pendulum sliding down an incline (20 points). A pendulum, which consists of a massless string of length l attached to a mass m, swings from the top of a rigid mast, which is attached perpendicularly to a cart sliding down a frictionless inclined plane, for example, down an inclined airtrack attached rigidly to the tabletop, as shown in Figure 1. The cart M and the mass m are both released from rest. The inclined plane is inclined at an angle θ with respect to the horizontal, and the mass m of the pendulum is initially inclined at an angle θ' with respect to the mast. You may assume that the cart is sufficiently massive (i.e., that $M >> m$) that the entire assembly of cart plus pendulum undergoes *uniform* acceleration down the incline during the swinging motion of the pendulum. (HINT: Use Einstein's principle of equivalence, i.e., that *uniform* acceleration is equivalent to the application of an *effective* gravitational field.)

(a) What must the initial release angle θ' have to be so that the pendulum undergoes no swinging motion at all as the cart slides frictionlessly down the incline?

(b) For small angular disturbances away from the angle θ' which you have found in part (a), what is the period of oscillation of the pendulum as the cart slides frictionlessly down the incline?

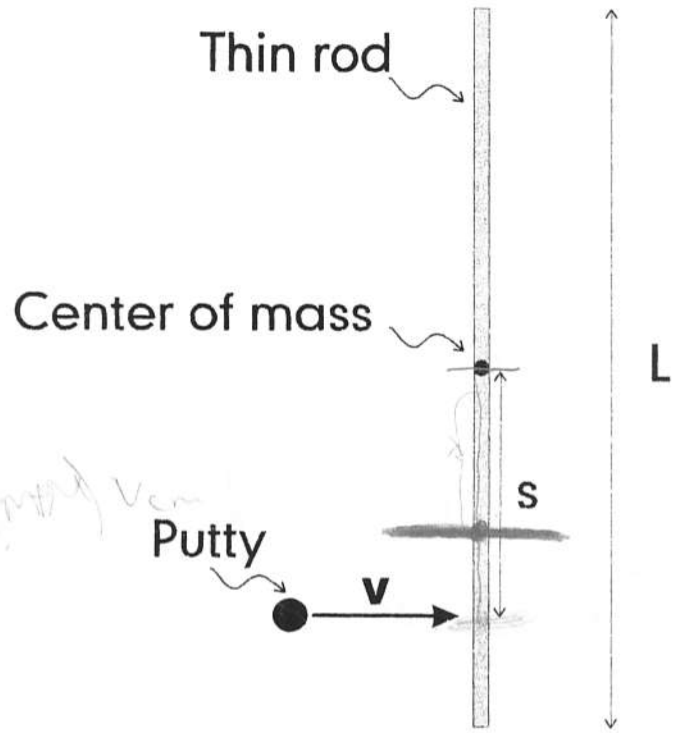

Figure 2: Figure for Problem 2.

PROBLEM 2: Collision of putty and rod on ice (20 points). A thin rod of mass M and length L is at rest on the frictionless surface of an ice-skating rink. Initially, the rod is suddenly struck at a distance s from its center of mass by a piece of putty of mass m, which is sliding along the surface of the ice, and which is initially travelling perpendicularly to the rod with a velocity $\mathbf{v}$. The collision between the putty and the rod is perfectly inelastic, and the putty sticks to the rod after this collision.

(a) What is the translational velocity $\mathbf{V}$ of the new center of mass of the system after the collision?

(b) What is the angular velocity ω of the system after the collision?

(c) How much heat energy is released during the collision?

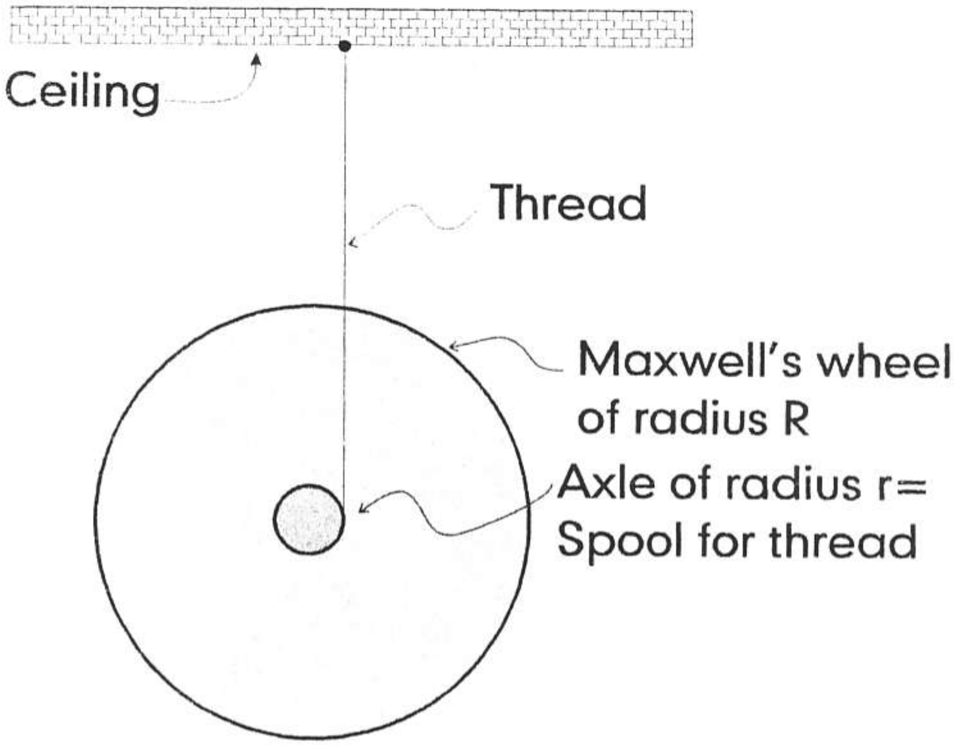

Figure 3: Figure for Problem 3.

PROBLEM 3: Maxwell-wheel Yo-Yo (20 points). A Maxwell's wheel consists of a solid cylinder of radius R and mass M joined solidly to a cylindrical, central axle of radius r and mass m (see Figure 3). The radius r is smaller than the radius R. This wheel hangs by means of a vertical thread from the ceiling in a Yo-Yo configuration as shown in Figure 3: The top of string is attached to the ceiling, and the bottom of the string is wrapped many times around the central axle of the wheel, which serves as a spool for the thread. Assume that the thread does not slip with respect to the surface of the axle during the wheel's motion. What is the downwards linear acceleration of the Maxwell's wheel due to gravity?

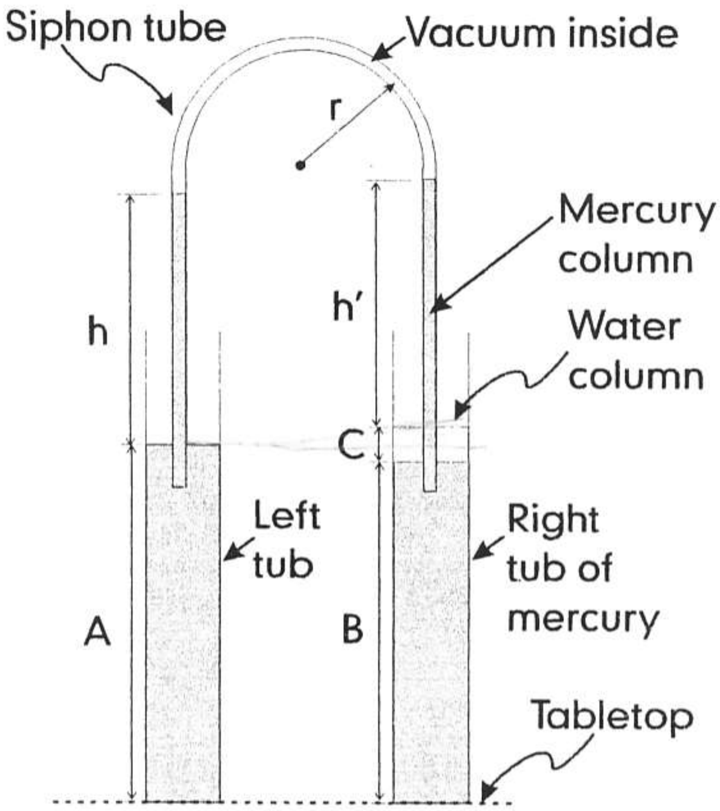

Figure 4: Figure for Problem 4.

PROBLEM 4: A siphon connecting two tubs (20 points). A siphon tube consists of glass tubing bent into a semicircle of radius $r = 760$ mm, and then joined at its two ends to two long, vertical straight sections of glass tubing (see Figure 4). The inside radius $a = 1.11$ mm of this tubing is much smaller than r. The siphon is initially inverted into a U-tube configuration, completely filled with liquid mercury, and then righted into a vertical configuration shown in Figure 4, so that its two openings are simultaneously immersed into two identical tubs of mercury, but at first without the water being present in the right tub. It is then observed that the mercury levels on both sides of the siphon then drop down to two equal levels $h' = h = 760$ mm, thus initially creating a vacuum in the upper, semicircular part of the siphon. The two identical tubs of mercury placed on the horizontal tabletop consist of two identical cylinders with an inside radius $b = 111$ mm which is much larger than a, with the mercury filling the left tub to a height A, and the right tub to a height B, with $A = B$ at first. Assume that A and B are both much larger than h and h'. Now water is poured into the right tub to a height of $C = 25.4$ mm above the right-hand mercury level. You are given that the density of mercury is 13.6×10^3 kg/m^3, and that the density of water is 1.00×10^3 kg/m^3.

(a) What is the height h' of the mercury level on the right side after the water has been poured into the right tub, and the system has come into equilibrium? Assume that the atmospheric pressure remains the same on the left and right sides, and that water and mercury are immiscible liquids. Assume that the mercury and the water are incompressible.

(b) For small disturbances, what are the oscillation periods of the two liquid mercury levels on the left and right sides of the siphon as shown in Figure 4? Derive algebraic expressions for these two periods, and evaluate numerically to three significant figures. Since $b >> a$, we can neglect changes in heights of the fluids in the tubs. Consider separately small disturbances on the left and on the right, and comment on whether these oscillation periods differ.

(c) Now lower vertically the siphon tube deeper into the two tubs, so that at a certain depth of immersion, the vacuum in the top, semicircular part of the siphon just barely disappears. For small disturbances, what is the new oscillation period of the entire liquid system after the vacuum has just barely disappeared? Derive an algebraic expression for this period, and evaluate numerically to three significant figures.

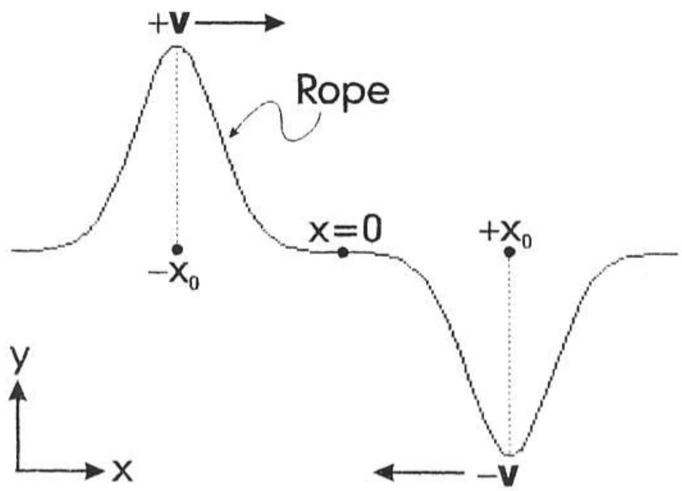

Figure 5: Figure for Problem 5.

PROBLEM 5: Two colliding Gaussian pulses on a rope (20 points). A snapshot at $t = 0$ of two counter-propagating Gaussian pulses on a disturbed rope is shown in Figure 5. The rope is very long, has a uniform linear mass density of μ, and is stretched out with a uniform tension T. At equilibrium before it was disturbed, the rope was completely straight, and coincided with the x axis. But now the displacement of the rope at $t = 0$ is given by

$$y(x, t = 0) = A\left\{\exp\left(-\frac{1}{2}\left(\frac{x + x_0}{a}\right)^2\right) - \exp\left(-\frac{1}{2}\left(\frac{x - x_0}{a}\right)^2\right)\right\}$$

which describes a superposition of two Gaussian pulses, as shown in Figure 5. The pulse peaked at $-x_0$ is travelling to the right, and the pulse negatively peaked at $+x_0$ is travelling to the left, so that they will eventually collide at the center of the rope located at $x = 0$. Assume that x_0 is much larger than a.

(a) What does this solution to the wave equation for the rope become at any arbitrary time t?

(b) At what later time $t = t_{coll}$ do the two Gaussian pulses collide, such that that their respective maximum and minimum coincide at $x = 0$? Express your result in terms of x_0, T, and μ. Find the solution $y(x, t = t_{coll})$, and sketch a snapshot of what the rope would look like at this instant of time.

(c) Where did the energy of the two pulses go at $t = t_{coll}$? Show from part (a) that energy is conserved when $t \to \pm\infty$, and also for the case of $t = t_{coll} + \varepsilon$, where ε is small. (HINTS: The energy of a pulse is $E_{pulse} = \int_{-\infty}^{+\infty} dx\, \mu\, (\partial y/\partial t)^2$. For a small ε', $\exp(\varepsilon') \approx 1+\varepsilon'$. Also, since $I(\alpha) \equiv \int_{-\infty}^{+\infty} \exp(-\alpha x^2)\, dx = \sqrt{\pi/\alpha}$, then $-dI(\alpha)/d\alpha = \int_{-\infty}^{+\infty} x^2 \exp(-\alpha x^2)\, dx = \sqrt{\pi/4\alpha^3}$).