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Math54 Midterm I

This is a closed everything exam, except a standard one-page cheat sheet (on one-side only). You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Problems are not necessarily ordered according to difficulties. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	5	
2	19	
3	19	
4	19	
5	19	
6	19	
Total	100	

1. (5 Points) Write your personal information below.

   Your Name: ______________________________

   Your GSI: ______________________________

   Your SID: ______________________________

2. (19 Points) Show that you need at least m vectors to span a linear space of dimension m.

3. (19 Points) Find all invertible $n \times n$ matrices A such that $A^2 + A = 0$.

4. (19 Points) Consider a linear system of equations $A\,x = b$, where

$$A = \begin{pmatrix} 0 & k & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 1+k \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}.$$

- For which values of k does the system have a unique solution and what is it?

- For which values of k does the system have no solution?

- For which values of k does the system have infinite number of solutions and what are they?

5. (19 Points) Let V be a subspace in $\mathcal{R}^3$. Show that there exists a 3×3 matrix A such that $V = \mathbf{im}(A)$.

6. (19 Points) Let $\mathcal{P}$ be the set of all polynomials, i.e.,

$$\mathcal{P} = \{\alpha_0 + \alpha_1 x + \cdots + \alpha_n x^n, \ \text{ where } \ \alpha_0, \alpha_1, \cdots, \alpha_n \in \mathcal{R}, \ \text{ and } n \geq 0 \text{ is any integer.}\}$$

Define a linear transformation $T : \mathcal{P} \to \mathcal{P}$ as

$$T\left(f(x)\right) = x\, f(x),$$

for any $f(x) \in \mathcal{P}$.

(a) Find $\mathbf{ker}(T)$.

(b) Find a polynomial $g(x)$ that is in $\mathcal{P}$ but not in $\mathbf{im}(T)$.

(c) Find a basis for $\mathbf{im}(T)$.