## E7 Midterm Exam 2

NAME :
SID :
SECTION : $\mathbf{1}$ or $\mathbf{2}$ (please circle your discussion section)
LAB

| \#11: TuTh 8-10 | \#12: TuTh 10-12 | \#13: TuTh 12-2 | \#14: TuTh 2-4 |
| :--- | :--- | :--- | :--- |
| \#15: TuTh 4-6 | \#16: MW 8-10 | \#17: MW 10-12 | \#18: MW 2-4 |
| \#19: MW 4-6 | \#20: TuTh 10-12* | \#21: MW 3-5 | \#22: TuTh 4-6 |

(please circle your lab section) * in Wheeler

| Problem | Points | Points |
| :---: | :---: | :---: |
| 1 | 26 |  |
| 2 | 6 |  |
| 3 | 22 |  |
| 4 | 20 |  |
| 5 | 24 |  |
| 6 | 12 |  |
| TOTAL | $\mathbf{1 1 0}$ |  |

Carefully read and follow these instructions:

1. Write your name on the top right corner of each page.
2. Start answering the exam only when instructed to do so.
3. Record your answers only in the spaces provided.
4. You may not ask questions during the exam.
5. You may not leave the exam room before the exam ends.
6. You may not use any electronic devices.
7. You may use two $8.5 \times 11$ sheets ( 4 pages) of handwritten notes.
8. Count the number of pages before the start of the exam.

There should be $\mathbf{1 6}$ pages.

1. Assume that the MATLAB script shown immediately below has been executed.
```
S(1).name = 'Bob'; S(1).age = 7;
S(2).name = 'Karen'; S(2).age = 8;
S(3).name = 'Michael'; S(3).age = 8;
S(4).name = 'Judy'; S(4).age = 7;
S(5).name = 'Jeff'; S(5).age = 7;
S(6).name = 'Linda'; S(6).age = 9;
```

(a) (2 pts)

On the space provided, write the output of the following MATLAB commands
>> class(S)
ans $=$ $\qquad$
>> size(S)
ans $=$ $\qquad$

Assume that the MATLAB script shown below has been executed.

```
T = S(1);
U = S(3:end);
```

(b) ( 4 pts )

On the space provided, write the output of the following MATLAB commands
>> class(T)
ans $=$ $\qquad$
>> size(T)
ans $=$ $\qquad$

```
>> class(U)
ans =
```

$\qquad$

```
>> size(U)
ans =
```

$\qquad$
(continues on the next page)
(c) (6 pts)

On the space provided, write the output of the following MATLAB commands
>> [T.name]
ans $=$ $\qquad$
>> [U.name]
ans $=$ $\qquad$
>> [U.age]
ans $=$ $\qquad$
(d) (4 pts)

Write a two-line MATLAB script, which when executed, will add by direct assignment another element to the end of S, with name Mary and age 13
$\qquad$
$\qquad$
$\qquad$
(e) (4 pts)

Write a two-line MATLAB script, which when executed, will add by direct assignment a field, named Height to 'Bob' and 'Karen' in S by using the string '5 ft 8 in' for 'Bob', '5 ft 3 in'
for 'Karen'.
$\qquad$
$\qquad$
$\qquad$
(f) (6 pts)

Using the MATLAB function upper ${ }^{1}$, write a three-line MATLAB script, which when executed, will capitalize all the names contained in S. Assume that all scripts above have been executed.
$\qquad$
$\qquad$
$\qquad$
2. Assume that the MATLAB script shown below has been executed.

```
x = 0:pi/5:pi;
y = sin(2.*x);
xi = 0:pi/100:pi;
```

[^0](a) (2 pts)

Fill in the blanks in the Matlab statement below, employing linear interpolation of y over x to determine array yi based on array xi ${ }^{2}$.

(b) (4 pts)

Complete the plot shown below, resulting from the execution of the MATLAB statement:

```
plot(x, y, 'o',xi,yi)
```

Assume that arrays $\mathrm{x}, \mathrm{y}$ xi and yi have been generated as the result of the MATLAB commands shown above, including the correct answer to part (a).


Figure 1: Figure resulting from the execution of $\mathrm{plot}(\mathrm{x}, \mathrm{y}, ~$ ' 0 ', $\mathrm{xi}, \mathrm{yi}$ )
-

[^1]3. The equations
\[

$$
\begin{aligned}
x_{1}+2 x_{2}+3 x_{3} & =6 \\
-x_{1}+4 x_{3} & =-13 \\
x_{2}+3 x_{3} & =-3 \\
x_{1}+3 x_{2}+x_{3} & =14
\end{aligned}
$$
\]

can be written in the form of

$$
\begin{equation*}
A x=b \tag{1}
\end{equation*}
$$

where,

$$
x=\left[\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right]
$$

(a) (6 pts)

Write below MATLAB expression to define the arrays $A$ and $b$ :

```
>> A =
```

>> b =
(b) (4 pts)

Assume that the above two commands have been executed and have produced the correct answers. Write down the output of the following lines of code.

```
>> size(A)
```

ans =
$\qquad$
>> size(b)
ans =
(continues on the next page)
(c) Consider now the least squares solution to Eq. (1):

$$
\begin{equation*}
\min _{x}\|A x-b\| \tag{2}
\end{equation*}
$$

where $\|e\|$ is the Euclidean norm of the vector $e$.

- Define $x^{*}$ as a solution to equation (2) and

$$
\begin{equation*}
c^{*}=\left\|A x^{*}-b\right\| . \tag{3}
\end{equation*}
$$

- Assume that the following MATLAB command has been executed for the correct values of $A$ and $b$ :
>> $\operatorname{rank}([\mathrm{A}, \mathrm{b}])$
ans =
4
i. (6 pts)

Write two MATLAB lines of code in order to respectively compute a least squares solution $x^{*}$ to Eq. (2) and $c^{*}$ as defined in Eq. (3):
>> xstar =
>> cstar =
ii. (6 pts)

Given the information provided, circle the statements below that are true.
the least squares solution $x^{*}$ is unique
$c^{*}=0$
an exact solution to Eq. (1) exists
the least squares solution $x^{*}$ is not unique $c^{*}>0$
an exact solution to Eq. (1) does not exist
4. Assume that the function myloop shown below can be executed. Write down the output of the following MATLAB commands.
(a) (10 pts)
>> flag $=$ myloop([1 29376548$])$
flag = $\qquad$
(b) (10 pts)
>> flag $=\operatorname{myloop}\left(\left[\begin{array}{lllllllll}7 & 8 & 4 & 3 & 1 & 2 & 6 & 7 & 9\end{array}\right]\right)$
flag = $\qquad$

```
function flag = myloop(a)
```

flag = 0;
k=1;
while ( (k <= length(a)) \& (flag ==0) )
$j=1$;
while ( (j<=k-1) \& (flag ==0) )
if (a(k) =a $a(j))$
flag = flag + 1;
end
$j=j+1 ;$
end
$\mathrm{k}=\mathrm{k}+1$;
end
5. (24 pts)

In combinatorial mathematics, the Catalan numbers form a sequence of natural numbers that satisfy the recursive relation

$$
\begin{align*}
C_{0}=1, \quad C_{n} & =\sum_{i=0}^{n-1} C_{i} C_{n-1-i} \quad n>0  \tag{4}\\
& =C_{0} C_{n-1}+C_{1} C_{n-2}+\cdots+C_{n-1} C_{0}
\end{align*}
$$

where $n$ and $i$ are integers.
The Catalan numbers for $n=0,1,2,3,4$, and 5 are respectively $1,1,2,5,14$, and 42 .

Complete the four missing or incomplete lines of the function CatR, which computes the nth Catalan number $C_{n}$ using the recursive relation in Eq. (4).

> begin code
function C $=\operatorname{CatR}(\mathrm{n})$
\% Computes the nth Catalan number Cn
\% n is a non-negative integer
if
-----------------------------------------------------------------
C = 1;
else

for

$\mathrm{C}=$

end
end
6. Assume that the MATLAB script shown below has been executed.

```
x = [1 3 5];
y = [3 7 15];
p = [2 3 6];
```

Write down the output of the following lines of code. If the code returns an error, write ERROR.
(a) ( 2 pts$)$

```
>> size(diff(x))
```

ans $=$
(b) ( 4 pts$)$
>> diff(y)./diff(x)
ans $=$
$\qquad$
(c) ( 4 pts$)$
>> polyval(p,[-1 2])
ans =
$\qquad$
$\qquad$
(d) ( 2 pts )
>> polyder (p)
ans =


[^0]:    ${ }^{1} \mathrm{t}=$ upper ('str') converts any lowercase characters in the string str to the corresponding uppercase characters and leaves all other characters unchanged.

[^1]:    ${ }^{2} \mathrm{Vq}=$ interp1 $(\mathrm{X}, \mathrm{V}, \mathrm{Xq}, \mathrm{METHOD})$ interpolates to find Vq , the values of the underlying function $\mathrm{V}=\mathrm{F}(\mathrm{X})$ at the query points Xq.

