University of California, Berkeley
Physics 110B Spring 2004 (*Strovink*)

EXAMINATION 1

Directions: Do all three problems, which have unequal weight. This is a closed-book closed-note exam except for Griffiths, Pedrotti, a copy of anything posted on the course web site, and anything in your own original handwriting (not Xeroxed). Calculators are not needed, but you may use one if you wish. Laptops and palmtops should be turned off. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Show all your work. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

Problem 1. (35 points)
A particle traveling with respect to the lab frame with velocity $\beta c\hat{x}^1$ has a physical property represented by the contravariant Lorentz four-vector h^μ. It is known that $p_\mu h^\mu = 0$, where p_μ is a component of the particle's four-momentum (expressed in covariant form). (As usual, repeated indices are summed.)

Denote by h'^μ the components of h as viewed in the rest frame of the particle. Based on the information given above, some of the h'^μ could be nonzero.

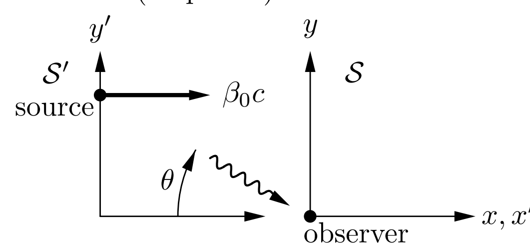
(**a.**) (15 points)
Can you tell whether h is timelike? Explain.

(**b.**) (20 points)
As a function of β and of those components of h' which could be nonzero, calculate all four components of h in the lab frame.

Problem 2. (30 points)
In a pair annihilation experiment, a positron (mass m) with total energy $E = \gamma mc^2$ hits an electron (same mass, but opposite charge) at rest. The two particles annihilate, producing two photons. If one of the photons emerges at angle θ relative to the incident positron direction, show that its energy ϵ is given by

$$\frac{mc^2}{\epsilon} = 1 - \sqrt{\frac{\gamma-1}{\gamma+1}}\cos\theta \ .$$

Problem 3. (35 points)



Consider Lorentz frames $\mathcal{S}$ and $\mathcal{S}'$, with spatial origins coincident at $t = t' = 0$. As usual, frame $\mathcal{S}'$ moves in the $\hat{x} = \hat{x}'$ direction with velocity $\beta_0 c$ relative to frame $\mathcal{S}$. A wave is emitted by a source that is at rest with respect to $\mathcal{S}'$. As seen by an observer who is at rest at the origin of the lab frame $\mathcal{S}$, the wave shown in the figure travels with phase velocity $\beta_{\rm ph} c$ at an angle θ with respect to the $\hat{x}$ direction ($\theta = 0$ if directly approaching, $\theta = \pi$ if directly receding). However, as seen by an observer who is at rest with respect to the frame $\mathcal{S}'$, show that the wave makes a different angle θ' with respect to the $\hat{x}'$ direction, where

$$\tan\theta' = \frac{\sin\theta}{\gamma_0(\cos\theta - \beta_0\beta_{\rm ph})} \ .$$