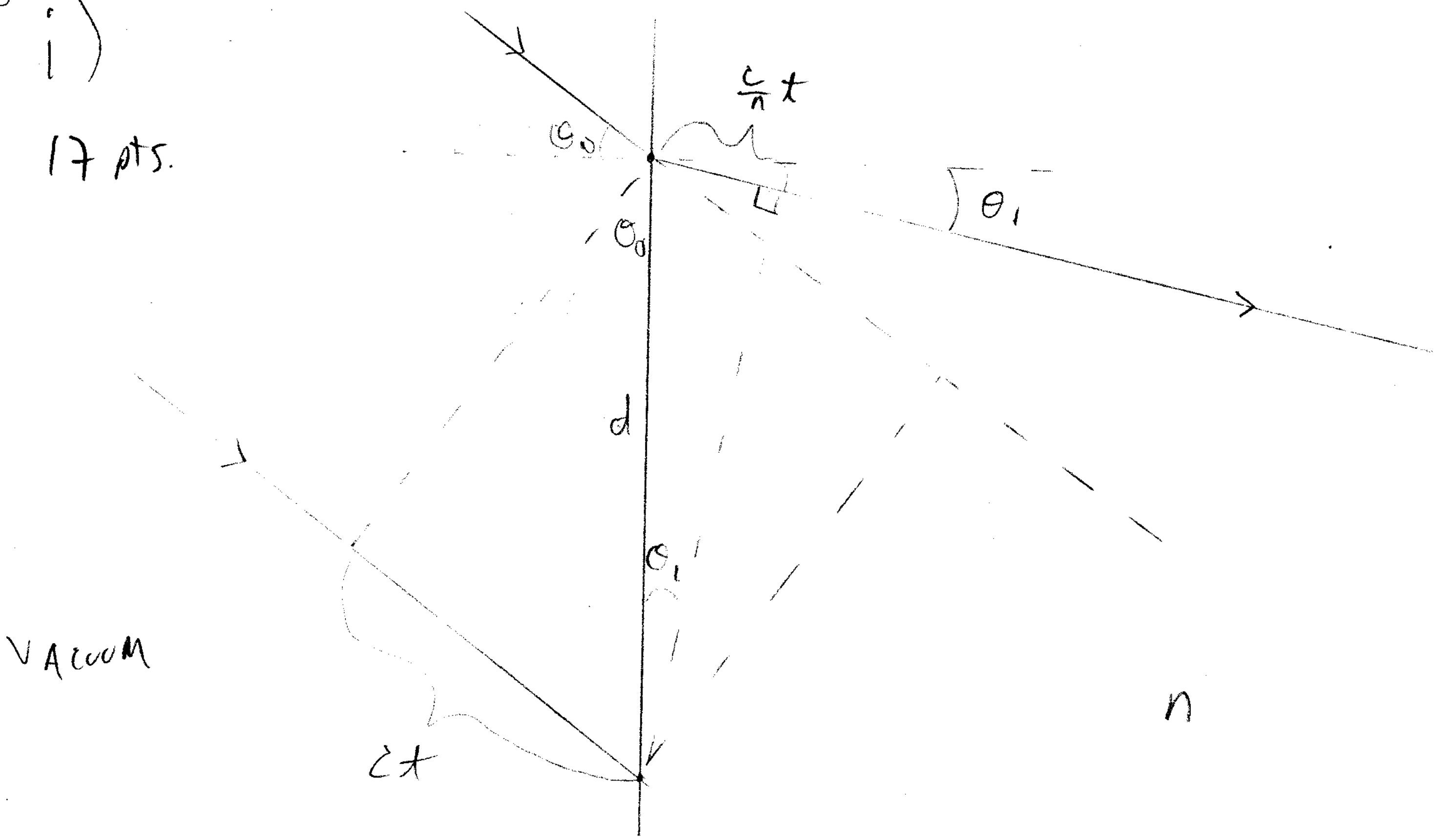
7C S05 BATTAGLIA FINAL EXAM SOLS.

A1

i)

17 pts.



TIME LAG BETWEEN WAVELETS STAYS FIXED

$$ct = d\sin\theta_0 \qquad \frac{c}{n}t = d\sin\theta_1$$

$$\Rightarrow \boxed{\sin\theta_0 = n\sin\theta_1} \quad ✓$$

ii) 8 pts.

$$E = \gamma mc^2 \Rightarrow \gamma = \frac{150}{105} = \frac{10}{7}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{0.51} \simeq 0.714$$

$$\rightarrow \boxed{n = \frac{1}{\beta} \simeq 1.4}$$

- ALCOHOLS (PURE)
- SOLID Kr; LIQUID Xe
- SALT CRYSTALS

$\underbrace{\qquad\qquad}_{2\text{ pts}}$

A2 (25 pts.)

$$\phi = \frac{hc}{\lambda_{max}} = \frac{1240\ eV \cdot nm}{683\ nm} = 1.82\ eV$$

WORK FUNCTION

$$= 2.91e{-19}\ J$$

10 pts.

SPACING BETWEEN ELECTRONS $\sim a_0$

AREA / e^- $\simeq \pi a_0^2$

ENERGY/AREA $\simeq \frac{\phi}{\pi a_0^2}$ FOR PHOTOELECTRONS

$$= \frac{2.91e{-19}\ J}{\pi \cdot (1e{-10}\ m)^2} = 9.26\ \frac{J}{m^2} \equiv \mathcal{E}$$

10 pts

$$I = \frac{15\ mW}{(1e{-3})^2} = 15000\ \frac{W}{m^2}$$

$$\mathcal{E} = I t \Rightarrow t = \frac{\mathcal{E}}{I} = \frac{9.26\ J/m^2}{1.5e4\ W/m^2}$$

$$\boxed{t \simeq 0.6\ ms}$$

5 pts.

B1

i) ENERGY MOMENTUM

8 pts $E_+ + m_- c^2 = E_Z$ $p_+ = p_Z$

$p_+^2 = p_Z^2$

$E_+^2 - m_e^2 c^4 = (E_+ + m_e c^2)^2 - m_Z^2 c^4$

$\rightarrow \cancel{E_+^2} + (m_Z^2 - m_e^2)c^4 = \cancel{E_+^2} + 2 m_e c^2 E_+ + m_e^2 c^4$

$\rightarrow E_+ = \dfrac{(m_Z^2 - 2m_e^2)c^4}{2 m_e c^2} \simeq \boxed{10^9 \text{ GeV}}$

ii) $2E_0 = m_Z c^2 \rightarrow E_0 = \dfrac{m_Z c^2}{2} = \boxed{500 \text{ GeV}}$

4 pts

iii) #1: MINIMUM WHEN THEY COINCIDE

8 pts. 4 pts. $P_Z > 200 \text{ GeV}/c \Rightarrow E_Z > \sqrt{200^2 + 1000^2}$

$> 1020 \text{ GeV}$

$10^9 \text{ GeV} \gtrsim 1020 \text{ GeV}$

NEXT →

4 pts. #2: E_0 500 GeV $\Rightarrow p = \sqrt{(500 \text{ GeV})^2 - (0.5 \text{ MeV})^2}/c$

$p \simeq 500 \text{ GeV}/c > 100 \text{ GeV}/c$

$\boxed{> 100 \text{ GeV}/c \text{ PARTICLES ALWAYS PRODUCED}}$

iv) $F = qvB = \frac{m\gamma v^2}{R}$

5pts.

$$\rightarrow R = \frac{m\gamma v}{qB} = \frac{p}{qB}$$

$$= \frac{100\ \mathrm{GeV}/c}{(1e)\cdot 3T} = \frac{10^{11}\ V}{3T}\cdot\frac{1}{3e8\ m/s}$$

$$\boxed{R \simeq 100\ m}$$

B2 (25 pts) $p \longrightarrow \quad \rightsquigarrow \gamma$

WE CAN GUESS THAT THE INITIAL PROTON ENERGY, E_0, WILL BE ENORMOUS. THEREFORE WE MAKE THE APPROXIMATIONS:

$$p_0 \simeq \frac{E_0}{c}$$

ALSO: AT THRESHOLD, WE USE THE INVARIANT MASS, AS THE p & π ARE "BARELY CREATED" IN THE CENTER-OF-MOMENTUM FRAME:

$$(E_0 + E_\gamma)^2 - (E_0 - E_\gamma)^2 = (m_p + m_\pi)^2 c^4$$

$$\cancel{E_0^2} + 2E_0E_\gamma + \cancel{E_\gamma^2}$$

$$- \cancel{E_0^2} + 2E_0E_\gamma - \cancel{E_\gamma^2} =$$

$$\rightarrow E_0 = \frac{(m_p + m_\pi)^2 c^4}{4E_\gamma} \simeq 9.7\text{e}14 \text{ MeV}$$

$$\rightarrow \boxed{E_0 \sim 10^{14} \text{ MeV} = 10^{20} \text{ eV}} \quad (\text{CORRECT: } 5 \times 10^{19} \text{ eV})$$

Rubric - C1, C2, C3; (25 pts. each)

C1) 17/25 for method 1 (see sol'ns)

-5 pts. for not using uncertainty principle

C2)

i.) 8 pts. (3 for $U(r)$, 5 for Schrödinger eqn.)
(-2 for $\frac{\partial^2}{\partial r^2}$, instead of $\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}$)

ii.) 8 pts. ~~(-2 for $\frac{\partial^2}{\partial r^2}$, instead of $\frac{\partial}{\partial r} r \frac{\partial}{\partial r}$)~~

iii.) 9 pts.

C3)

i.) 5 pts.

ii.) 10 pts.

iii.) 10 pts.

C1] $\Delta p \, \Delta x \geq \hbar/2$

Method 1: use non-relativistic momentum: $p = mv$

$$\Delta x = L \quad \Rightarrow L \geq \frac{\hbar}{2mv} = \frac{\hbar}{2m\beta c}$$

$\cancel{\Rightarrow \Delta v \geq \frac{\hbar}{2mL}}$ $\quad m_{rel} = \gamma m_{rest} = 1.05\, m_{rest}$

$$\Rightarrow \gamma = 1.05$$

$$\Rightarrow \beta = \sqrt{1 - 1/\gamma^2} = .3$$

$$\Rightarrow L = \frac{\hbar c}{.3(2mc^2)} \cong \frac{1240 \text{ eV}\cdot\text{nm}}{2\pi(.6)(.511\times10^6 \text{ eV})} = \cancel{.004 \text{ nm} = 4 \text{ pm}}$$

$$= .64 \text{ pm}$$

Method 2 (more accurate): relativistic momentum $p = \gamma mv$

$$\Rightarrow L \geq \frac{\hbar}{2\gamma mv} = \frac{\hbar c}{2\gamma\beta(mc^2)} \cong \frac{1240 \text{ eV}\cdot\text{nm}}{(2\pi)2(1.05)(.3)(.511\times10^6 \text{ eV})} = .00385 \text{ nm} = 3.85 \text{ pm}$$

$$= .613 \text{ pm}$$

C2

i.) $U(r) = -\frac{ke^2}{r}$

$$-\frac{1}{r^2}\frac{\hbar^2}{2\mu}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi(r,t)}{\partial r}\right) + U(r)\psi(r) = i\hbar\frac{\partial\psi(r,t)}{\partial t},$$

or

$$-\frac{1}{r^2}\frac{\hbar^2}{2\mu}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi(r)}{\partial r}\right) + U(r)\psi(r) = E\psi(r)$$ for energy eigenfunctions,

where

$$\mu = \frac{m_e^2}{m_e+m_e} = \frac{m_e}{2} \quad (m_{e-} = m_{e+} \equiv m_e)$$

ii.) $\psi_{100} = \frac{1}{\sqrt{\pi a^3}}e^{-r/a}$, where $a \equiv \frac{\hbar^2}{\mu ke^2} = 2a_0$

iii.) $P(r) = 4\pi r^2|\psi(r)|^2$ $\cancel{|\psi_{100}(r,\theta,\phi)|}$

<u>For H,</u> $P(r) = 4\pi r^2\left(\frac{1}{\pi a_0^3}\right)e^{-2r/a_0}$

$$\Rightarrow 0 = \frac{dP}{dr} = \frac{8r}{a_0^3}e^{-2r/a_0} - \frac{8r^2}{a_0^4}e^{-2r/a_0}$$

$$= \frac{8r}{a_0^3}e^{-2r/a_0}\left(1-\frac{r}{a_0}\right)$$

$\Rightarrow$ <u>$r_{most\text{-}probable} = a_0$ for H</u>

<u>For Positronium,</u> $a = 2a_0 \Rightarrow$ <u>$r_{most\text{-}probable} = 2a_0$</u>

(3)

i.) $U = +\frac{2(Z-2)e^2 k}{r}$

$\cong \frac{+2(98)(1.440\ \text{eV}\cdot\text{nm})}{9\ \text{fm}}$

$\cong +31.36\ \text{MeV}$

ii.) $\alpha a \equiv \sqrt{\frac{2m}{\hbar^2}(V-E)}\; a = \sqrt{\frac{8m_p}{\hbar^2}(31.36\ \text{MeV} - 9\ \text{MeV})}\;(10\ \text{fm})$

$= \frac{\sqrt{8(938\ \text{MeV})(22.36\ \text{MeV})}}{(1240\ \text{eV}\cdot\text{nm})/2\pi}\,(10\ \text{fm})$

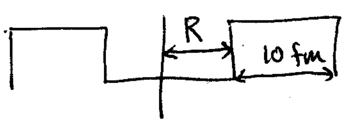
$= \cancel{33}\ 20.76$

$\Rightarrow \alpha a \gg 1$, ~~but $\alpha a \not> 1$~~

$\Rightarrow T = \left[1 + \frac{\sinh^2 \alpha a}{4\left(\frac{E}{V}\right)\left(1-\frac{E}{V}\right)}\right]^{-1} = \cancel{.004422}\ 3.6\times10^{-18}$

~~Pretending~~ We have that $\alpha a \gg 1$, we can use

$T \cong e^{-2\alpha a}\left[16\,\frac{E}{V}\left(1-\frac{E}{V}\right)\right] = \cancel{.004154}\ 3.07\times10^{-18}$ (close enough)

iii.) 

$R = 9\ \text{fm}$; non-relativistic $\Rightarrow E \cong \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{E}{2m_p}} = \sqrt{\frac{9\ \text{MeV}\cdot c^2}{2(938\ \text{MeV})}} = .07c$

$\frac{\text{prob.}}{\text{s}} = \frac{\text{reflections}}{\text{s}} \times \frac{\text{prob.}}{\text{refl.}} \cong \frac{v}{2R}\cdot T$

$\Rightarrow \tau \cong \frac{1}{\left(\frac{v}{2R}\cdot T\right)} = \frac{2(9\ \text{fm})}{(.07c)(\cancel{.004422}\ 3.6\times10^{-18})} \approx \cancel{10^{-21}\ \text{s}}\ \cancel{2\times10^{-14}\ \text{s}} \approx 2.4\times10^{-4}\ \text{s}$