Print your name: Caleb Boyd Signature:



Discussion Section: 206

Math 54 Second Midterm Fall 2012

Instructor: D.-V. Voiculescu

This is a "closed book" exam, so you may not bring in or use notes or the textbook.

Calculators are not allowed.

Please write your name, SID and Discussion Section # on everything you hand in, including this sheet of paper on which you have to provide the answer to Problem II (the true or false questions). For Problem I you must show the method and calculations you use to get the answers (write the solutions to the questions in Problem I in your blue book). The Requirement is 20 points.

Problem I (3+2+2+3+3 pts) Let A and B be the matrices:

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 0 & 1 \end{pmatrix}$$

True | False

a) Find the eigenvalues and eigenvectors of A.

b) Find an invertible matrix S and a diagonal matrix D so that $D = S^{-1}AS$.

c) Apply Gram-Schmidt to the columns of B to find an orthonormal basis of Col (B).

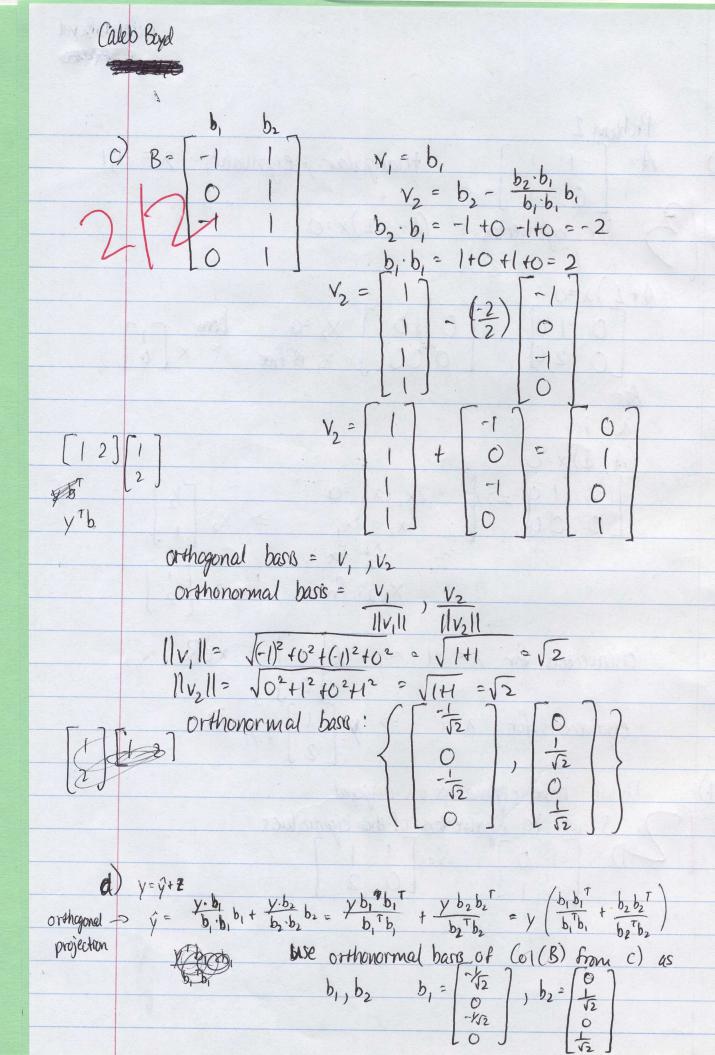
d) Find the 4x4 matrix of the orthogonal projection onto Col (B).

e) Find a least squares solution of Bx = $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

Problem II (7 pts, each question 1 pt). Check True or False.

	a) If a, b, c, d $\in \mathbb{R}$ then $\sqrt{a^2 + 4b^2} \sqrt{c^2 + d^2} > ac + 2bd $			
9	b) The inverse of an orthogonal matrix is an orthogonal matrix		Section Control of Con	and and an experience of the second
	c) $\langle \binom{a}{b}, \binom{c}{d} \rangle = \det \binom{a}{b}$ is an inner product on \mathbb{R}^2	*	\	
	d) If E, F are symmetric 2x2 matrices, then so is EF.	MA	\	False
4	e) If W is a subspace of TR and {x, y} and {z, t} are bases of W and W, then {x, y, z, t} is a basis of TR.	\	Command of the Comman	,
	f) An orthogonal 2x2 matrix is always diagonalizable	was the second of the second o	1	False
	g) If M is a 3x2 matrix then rank (A) + nullity ($A^{(1)}$) = 3	1		

	Problem 1		
2)			
2	For eigenvectors: (A- AI) X=0		
1	For eigenvectors: $(A-\lambda I)x=0$		
	$(A+I)\chi^2O$		
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	eigenvectors for $\chi = -1 => \chi / 1$, $\chi \in \mathbb{R}$		
	Reversion On the Hall of the state of the st		
	expensedors for $\lambda=1 \Rightarrow \chi(1), y \in \mathbb{R}$		
	[2]		
b)	D will have eigenvalues on diagonal		
0	S will be eigenvectors of those eigenvalues		
L	D= [-10] S= 11		
	[01] [02]		
	- Will did to a free to dead to be some		



$$\frac{d}{d} = \frac{b_1 \cdot b_2 = b_2 \cdot b_3 = 1}{b_1 \cdot b_2 = b_2 \cdot b_3 = 1}$$

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