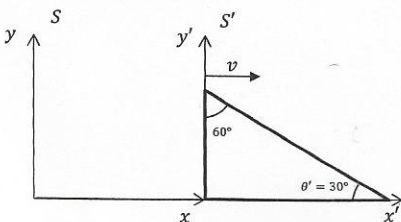


1. A region of space has the potential shown to the left.
  - a. In each of the three regions, write down the time-independent Schrodinger equation, and the wavenumber of a particle with energy  $E > V_2$ .
  - b. A stream of particles with energy  $E > V_2$  and amplitude,  $A_{in}$ , travels from  $+\infty$ , and collides with the barrier. Write down the wavefunction for the particles in each of the three regions in terms of  $A_{in}$ , and the necessary arbitrary constants. (You can choose the variables for the constants.)
  - c. Write down the boundary conditions for the wavefunctions, and use them to write down equations that relate the constants in each of the three regions to one another. *You do not have to solve these equations!*

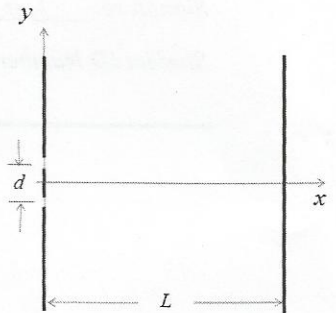
2. A particle with mass,  $m$ , is confined in a infinite potential well with width,  $L$ . The particle is in a mixed state, and its wavefunction is the linear combination

$$\Psi(x, t) = \frac{1}{\sqrt{L}} \left[ \sin \frac{\pi x}{L} e^{-i \frac{E_1}{\hbar} t} + \sin \frac{2\pi x}{L} e^{-i \frac{E_2}{\hbar} t} \right].$$

- a. What is the average energy,  $\langle H \rangle$ , of the particle in this state? Express your answer in terms of  $\hbar$ ,  $m$ , and  $L$ . Remember that  $H = \frac{p^2}{2m}$ , and the sum of two angles formulas for the integrals.
  - b. What is the uncertainty,  $\Delta H = \sqrt{\langle H^2 \rangle - \langle H \rangle^2}$ , in the energy of this particle? Express it in the same terms as part a.
3. Suppose that the hydrogen atom is held together not by the Coulomb force, but by the gravitational force with a potential energy,  $V(r) = -\frac{GmM}{r}$ . Namely, consider a light particle with mass,  $m_e$ , in orbit about a heavy particle,  $m_p \gg m_e$ , where both particles are neutral, and the only force acting between them is gravitational force. Using the Heisenberg uncertainty principle, estimate the energy of the orbiting particle in terms of  $\hbar$ ,  $G$ ,  $c$ ,  $m_e$ , and  $m_p$ .
  4. In the Compton effect where a photon scatters off an electron at rest, consider the case where the angle of the final photon  $\theta = \pi$ . Find the wavelength,  $\lambda_e$ , of the *electron* after the collision in terms of the wavelength,  $\lambda$ , of the incident light,  $\hbar$ ,  $c$ , and  $m_e$ .
  5. When light is used in the double-slit experiment to the right, the first minima in the intensity of light occurs at  $y_1$ . What must the relativistic energy of the electrons be if they were used instead, and if the first minima of the intensity of electrons is still at  $y_1$ ? Express your answers in terms of  $d$ ,  $L$ ,  $\hbar$ ,  $m_e$ ,  $c$  and  $y_1$ . You can make small angle approximations.



6. Dick is in a frame moving at a velocity,  $v$ , relative to Tom, who is in the stationary frame. In Dick's frame, there is a 60-30-90 triangle at rest relative to him (see figure to the left). Tom, on the other hand, sees this triangle as being 30-60-90 triangle with  $\theta = 60^\circ$  now. What is  $v$ ?



7. When an object is placed a distance,  $s$ , from a concave, reflective lens, the image is also at  $s$ .
  - a. The object is then moved to a distance,  $d_0 = s/4$ , from the mirror. What is  $d_i$ , the position of the image?
  - b. Determine whether the image is virtual or real.