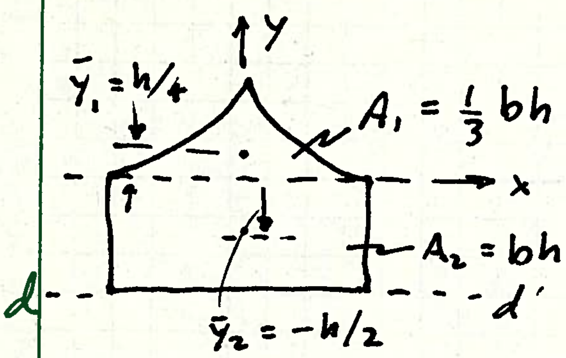


a) CENTROID OF COMPOSITE AREA

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i}$$

$$\bar{y} = \frac{(\frac{1}{3}bh)(h/4) + (bh)(-h/2)}{bh/3 + bh} = \frac{-5bh^2/12}{4bh/3}$$

$$\boxed{\bar{y} = -\frac{5}{16}h \quad (\text{BELOW THE X-AXIS})}$$

(b) $I_{dd'} = I_{1dd'} + I_{2dd'}$

USE PARALLEL AXIS THEOREM TO GET $I_{1dd'}$. FIRST WE NEED TO COMPUTE THE MOMENT OF INERTIA OF AREA 1 ABOUT ITS CENTROID.

$$\bar{I}_1 = I_{1x} - A_1\bar{y}_1^2 = \frac{1}{16}bh^3 - \left(\frac{bh}{3}\right)\left(\frac{h}{4}\right)^2$$

$$\bar{I}_1 = bh^3\left(\frac{1}{16} - \frac{1}{48}\right) = \frac{1}{24}bh^3$$

THEN

$$I_{1dd'} = \bar{I}_1 + A_1(\bar{y}_1 + h)^2$$

$$= \frac{1}{24}bh^3 + \left(\frac{1}{3}bh\right)\left(\frac{5}{4}h\right)^2 = bh^3\left(\frac{1}{24} + \frac{25}{48}\right)$$

$$I_{1dd'} = \frac{27}{48}bh^3$$

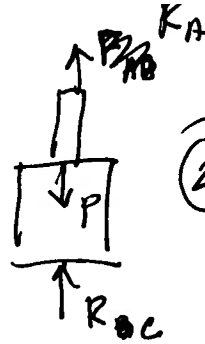
$$I_{2dd'} = I_{2x} = \frac{1}{3}bh^3 \Rightarrow \boxed{I_{dd'} = bh^3\left(\frac{1}{3} + \frac{27}{48}\right) = \frac{43}{48}bh^3}$$

35 pts

2-1

20 pts

a) FBD : Overall

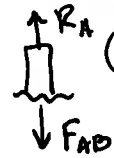


2

$\boxed{2}$

5 $\delta_A = \delta_{AB} + \delta_{BC} = \dfrac{F_{AB}L}{AE} + \dfrac{F_{BC}L}{2AE} = 0$

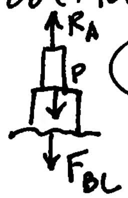
$\boxed{4}$ FBD : partial + Analysis
(A-B)



2

$F_{AB} = R_A$ 2

$\boxed{4}$ FBD : partial + Analysis
(B-C)



2

$F_{BC} = R_A - P$ 2

$\boxed{5}$

Overall Analysis: 2

$$\delta_A = \frac{R_A \cancel{L}}{\cancel{AE}} + \frac{(R_A - P)\cancel{L}}{2\cancel{AE}} = 0$$

$$R_A + \frac{R_A}{2} - \frac{P}{2} = 0$$

$$\frac{3R_A}{2} = \frac{P}{2} \Rightarrow R_A = \frac{P}{3} \uparrow$$

$\boxed{4}$

Other ΣF_y Reaction Analysis: 2

$$\Sigma F_y = R_A + R_C - P = 0$$

$$P/3 + R_C - P = 0$$

$$R_C = 2/3 P \uparrow$$

$\boxed{2\text{-}2}$

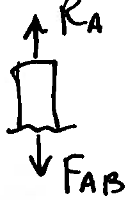
b) (5pts) det. shear τ in pin at top

$$\tau_{pin} = \frac{R_A}{2A_{pin}} = \frac{R_A}{2\frac{\pi}{4}d^2} = \frac{2R_A}{\pi d^2} \quad \text{OR} \quad \frac{2P}{3\pi d^2}$$

since pin is in double shear

c) (5pts) det axial σ's in A-B + B-C

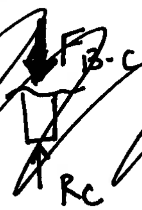
FBD: AB



$$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{R_A}{A}$$ tension

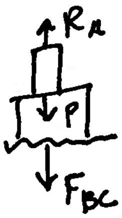
$F_{AB} = R_A$

~~FBD: B-C~~



$\cancel{\sigma_{BC} = \frac{F_{BC}}{2A} = \frac{R_C}{2A}}$

FBB: BC



$$\Sigma F_y = -F_{BC} + R_A - P = 0$$

$$F_{BC} = R_A - P$$
$$= -2/3 P$$

OR $F_{BC} = -R_C$

compression

$$\sigma_{BC} = \frac{F_{BC}}{2A} = \frac{-R_C}{2A}$$

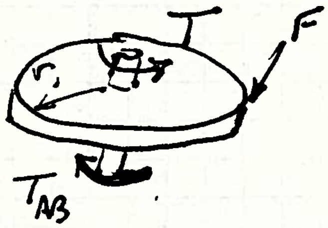
d) (5pts)

$$\delta_B = \delta_{AB} = \frac{F_{AB}L}{AE} = \frac{R_A L}{AE} = \frac{PL}{3AE} \downarrow$$

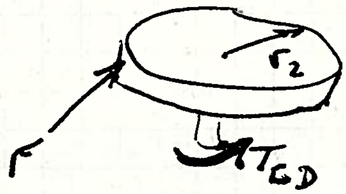
OR

$$\delta_B = \delta_{BC} = \frac{F_{BC}L}{2AE} = \frac{-R_C L}{2AE} = \frac{\cancel{2}PL}{3\cdot\cancel{2}AE} \downarrow$$

3. A KEY TO THIS PROBLEM IS A FREE BODY DIAGRAM. IT HELPS TO CLARIFY THE WAY THAT THE APPLIED TORQUE T IS DISTRIBUTED THROUGH THE GEARS TO THE TWO SHAFTS.



FBD OF GEAR 1



FBD OF GEAR 2

MOMENT EQUILIBRIUM OF GEAR 1: $T - T_{AB} - F r_1 = 0$ (1)

MOMENT EQUILIBRIUM OF GEAR 2: $T_{CD} - F r_2 = 0$ (2)

FROM (2), $F = \dfrac{T_{CD}}{r_2}$ (3)

FROM (1) & (3), $T = T_{AB} + \dfrac{r_1}{r_2} T_{CD}$ (4)

GEOMETRY: IF GEAR 1 ROTATES THROUGH AN ANGLE ϕ, A POINT ON ITS EDGE MOVES THROUGH A DISTANCE $r_1 \phi$. THE CORRESPONDING POINT ON GEAR 2 MOVES THROUGH THE SAME DISTANCE, SO

$$r_1 \phi = r_2 \phi_2$$ (5)

TORQUES T_{AB} & T_{CD} ARE DETERMINED BY KNOWING
$\phi_A = \phi$ AND $\phi_C = \phi_2 = \frac{r_1}{r_2}\phi$.

FOR A UNIFORM SHAFT WITH SHEAR MODULUS G, POLAR MOMENT OF INERTIA J AND LENGTH L, THE TORQUE-TWIST RELATION IS

$$T = \frac{GJ\phi}{L}$$

IN THIS PROBLEM BOTH SHAFTS ARE IDENTICAL, SO

$$J = \frac{1}{2}\pi r^4 = \frac{1}{2}\pi\left(\frac{d}{2}\right)^4 = \frac{1}{32}\pi d^4$$

SO THAT

$$T_{AB} = \frac{\pi d^4}{32L} G\phi \quad , \quad T_{CD} = \frac{r_1}{r_2}\frac{\pi d^4}{32L} G\phi \qquad (6)$$

USING EQ (6) IN (4) $\Rightarrow$

$$\boxed{T = \frac{\pi d^4}{32L} G\phi\left(1 + \left(\frac{r_1}{r_2}\right)^2\right)} \qquad (7)$$

COMMON ERRORS:

- NOT RECOGNIZING THAT T IS DISTRIBUTED BETWEEN THE TWO SHAFTS. IN THIS APPROACH, PEOPLE WROTE $T = T_{AB} = \frac{r_1}{r_2} T_{CD}$. IN DETERMINING T, TYPICALLY EITHER T_{AB} OR T_{CD} WAS USED AND THE RESULT WAS AS IF THE OTHER SHAFT DID NOT EXIST.

- ATTEMPTING TO ADD ANGLES AS WAS DONE IN SECTION 10.5 OF THE BOOK. IN THIS PROBLEM, THE ANGLE OF TWIST OF GEAR 1 IS GIVEN, SO EQ. (5) PROVIDES ALL OF THE GEOMETRIC INFORMATION NEEDED TO SOLVE THIS PROBLEM. NOTE THE DIFFERENCE BETWEEN THIS PROBLEM AND EXAMPLE 10.4, WHERE THE APPLIED TORQUE IS AT THE OTHER END OF THE SHAFT.

- DID NOT DRAW A COMPLETE FREE BODY DIAGRAM. AS NOTED ON THE FIRST PAGE, THIS REALLY HELPS IN SEEING HOW THE APPLIED TORQUE IS SHARED BETWEEN THE TWO SHAFTS.