Fall 2011

Midterm 2

NAME: Solution

SSID:

Instructions
Read all of the instructions and all of the questions before beginning the exam.

There are 4 problems in this exam. The total score is 100 points. Points are given next to each problem to help you allocate time. Do not spend all your time on one problem.

Unless otherwise noted on a particular problem, you must show your work in the space provided, on the back of the exam pages or in the extra pages provided at the back of the exam. Simply providing answers will only result in partial credit, even if the answers are correct.

Draw a BOX or a CIRCLE around your answers to each problem.
Be sure to provide units where necessary.

GOOD LUCK!

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"Pessimism is just an ugly word for pattern recognition."
-Anonymous

Problem 1 Quickie (12.5 points)

a) In the box below, provide a symbolic expression for \( v_o \) for \( t > 0 \) if \( v(0) = 4 \text{ V} \).

\[
V_o = -V(0) e^{-\frac{t}{RC}} \quad (V) \quad t > 0
\]

Ideal Op Amp: \( V_i = V_f = 0 \). \( \therefore \) No current into \( R_1 \).

\( I = C \cdot R_f \).

\( V_o(0) = 0 \). \( \therefore \) \( V_0(0) = -V(0) \).

\( \therefore \) \( V_0(t) = -V(0) e^{-\frac{t}{RC}} \quad (V) \)
b) If $R_f = 40 \, \text{k} \Omega$, $R_l = 10 \, \text{k} \Omega$, $C = 10 \, \mu\text{F}$, and $v(0) = 4 \, \text{V}$, write the expression for $v(t)$ for $t > 0$ in the BOX BELOW. \( (2.5 \text{ points}) \)

$$u_0(t) = -4e^{-t/40} \, \text{V} \quad t > 0 \, .$$

$$R_f \cdot C = 40 \, \text{k} \cdot 10 \, \mu\text{F} = 0.4$$
"If a person offends you... do not resort to extremes, simply watch your chance and hit him with a brick."
- Mark Twain

Problem 2 First order circuits

Consider the circuit below.

![Circuit Diagram]

a) What is the value of $V_o$ at $t = 0^+$? Write it in the BOX BELOW.

\[ V_o (0^+) = -9000 \, V. \]

\[ \dot{L} (0^-) = -1 \, A \implies \dot{I}_L (0^+) = -1 \, A. \]

\[ V_o (0^+) = \dot{L} (0^+). \]

\[ R_3 = -9000 \, V. \]
b) Using whatever method you like (yes, anything, don’t raise your hand to ask if you can use XXX), provide a **symbolic expression** for the voltage \( V_0(t) \) for \( t > 0 \) in the BOX BELOW.

\[
V_0(t) = -R_3 \ e^{-\frac{(R_1+R_2+R_3) t}{L}} \quad (V) \quad t > 0.
\]

\[
\mathcal{L} = \frac{L}{R_{eq}} = \frac{L_1}{R_1 + R_2 + R_3}.
\]

\[
V(0^+) = -R_3.
\]

\[
V(\infty) = 0.
\]

\[
V(t) = -R_3 \ e^{-\frac{t}{\mathcal{L}}}
\]
c) Using the values provided in the figure, provide an expression for the voltage $V_0(t)$ for $t > 0$ in the BOX BELOW. (2.5 points)

\[
V_0(t) = -9000 e^{-1.8 \times 10^6 t} \quad (V) \quad t > 0.
\]

\[
R_3 = 9000.
\]

\[
\frac{L_1}{R_1 + R_2 + R_3} = \frac{10 \text{ mH}}{3k + 6k + 9k} = \frac{5}{9} \times 10^{-6}
\]
"Accept that some days you are the pigeon, and some days you are the statue."
-David Brent, The Office

Problem 3 Second order circuits (40 points)

Consider the circuit below.

a) Find the second order differential equation for the variable \( \dot{i}_L \) that describes the circuit behavior for \( t > 1 \, \mu s \). Write into the box below. (15 points)

\[
\frac{d^2 i_L}{dt^2} + \left( \frac{1}{R_s C} + \frac{1}{R_1 C} + \frac{R_2}{L} \right) \frac{d i_L}{dt} + \frac{1}{L C} \left( \frac{R_2}{R_s} + \frac{R_2}{R_1} + 1 \right) i_L = \frac{1}{L C R_s}
\]

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b) Is there a value for R1 such that the resonant circuit can be critically damped for $t > 1 \, \mu s$? (5 points)

*Please show your equations clearly.*

If YES, write the value here: 

If NO, provide an expression that shows why not here:

$R_1 = -3 + 3\sqrt{2}$

$\alpha = \frac{1}{2} \left( \frac{R_3}{L} + \frac{1}{CR_3} + \frac{1}{CR_1} \right) = \frac{1}{2} \left( \frac{9k}{16\pi mH} + \frac{1}{\ln T \times 6k} + \frac{1}{\ln T \times \frac{R_1}{3}} \right) = \left( \frac{1}{3} + \frac{1}{2R_1} \right) \times 10^6$

$\omega_0 = \sqrt{\frac{1}{LC} \left( \frac{R_3}{R_3} + \frac{R_3}{R_1} + 1 \right)} = \frac{1}{\sqrt{\ln T \times \frac{gH}{6} + \frac{g}{R_1} + 1}} = \sqrt{\frac{1}{2R_1} + \frac{5}{36}} \times 10^6$

Critical damped $\alpha = \omega_0$.

$. \left( \frac{1}{3} + \frac{1}{2R_1} \right) \times 10^6 = \sqrt{\frac{1}{2R_1} + \frac{5}{36}} \times 10^6$

$\Rightarrow \frac{1}{9} + \frac{1}{3R_1} + \frac{1}{4R_1^2} = \frac{1}{2R_1} + \frac{5}{36}$

$\Rightarrow 4R_1^2 + 12R_1 + 9 = 18R_1 + 5R_1^2$

$\Rightarrow R_1^2 + 6R_1 - 9 = 0$

$\Rightarrow R_1 = \frac{-6 \pm \sqrt{36 + 36}}{2} = -3 + 3\sqrt{2}$
c) Determine the **two relevant** initial conditions of the circuit  

(5 points)

<table>
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<th>Condition 1:</th>
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<tbody>
<tr>
<td>$\dot{L}_L(1ms^+) = 0$</td>
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<table>
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<tr>
<th>Condition 2:</th>
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<td>$\dot{L}_L'(1ms^+) = 0$</td>
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\[ i_L(1ms^-) = 0 \implies i_L(1ms^+) = 0 \]

\[ V_C(1ms^-) = 0 \implies V_C(1ms^+) = 0 \]

\[ -i_L'(1ms^+) + i_L(1ms^+).R_2 = V_C(1ms^+) \]

\[ \implies \dot{L}_L'(1ms^+) = 0 \]
d) Assuming $R_1 = 9000 \ \Omega$, provide a complete expression for $i(t)$ for $t > 1 \ \mu s$.

\[ i_L(t) = \frac{1}{2I} + e^{-\frac{7}{18} \times 10^6 (t-1\mu s)} \left[ \frac{1}{2I} \cos \left( \frac{414}{18} \times 10^6 (t-1\mu s) \right) - \frac{1}{3I} \sin \left( \frac{414}{18} \times 10^6 (t-1\mu s) \right) \right] \ \text{mA} \]

\[ \omega_0 = \sqrt{\frac{5}{36} + \frac{1}{2R_1}} \times 10^6 = \frac{7}{18} \times 10^6 \]

\[ \omega^2 = \frac{7 \times 4}{18 \times 18} - \frac{7 \times 9}{36 \times 9} \times 10^{12} < 0 \quad \Rightarrow \quad \omega < \omega_0 \]

Underdamped.

\[ \omega_d = \sqrt{\omega_0^2 - \omega^2} = \sqrt{\frac{(6^2 - 4^2)}{18 \times 18} \times 10^{12}} = \frac{114}{18} \times 10^6 \]

\[ \dot{i}_L(\omega) = \frac{1}{2I} \cdot \frac{1}{6k + (9k / 1k)} = \frac{1}{21k} = \frac{1}{21} \ \text{mA} \cdot \text{s} \]

\[ \dot{i}_L(t) = \dot{i}_L(\omega) + e^{-\omega (t-\omega)} \left[ B_1 \cos \omega_d (t-\omega) + B_2 \sin \omega_d (t-\omega) \right] \]

\[ B_1 = -\dot{i}_L(\omega) = \frac{1}{21} \times 10^{-3} \]

\[ B_2 = \frac{i_L''(0) + \omega \left[ \dot{i}_L(0) - i_L''(0) \right]}{\omega_d} = -\frac{\frac{7}{18} \times 10^6 \times \frac{1}{3I} \times 10^{-3}}{\frac{114}{18} \times 10^6} = \frac{-1}{3I} \times 10^{-3} \]
"I know they were just kids... but man we beat the f$%^!$ out of them!"
- Dogma

Problem 4 Phasors

If $i_s = 5 \cos(10t + 40^\circ)$ A in the circuit below, find $i_o$

\[ i_o(t) = \frac{30\sqrt{2}}{19} \cos(10t + 85^\circ) \ A \]

\[ I_s = 5 e^{40^\circ} \]

\[ Z_1 = 4 + 2j \]
\[ Z_2 = 3 + \frac{1}{3j} \]

\[ I_0 = \frac{Z_1}{Z_1 + Z_2} \cdot I_s \]

\[ I_0 = \frac{\frac{8j}{4+2j}}{\frac{8j}{4+2j} + 3 + \frac{1}{3j}} \cdot 5e^{40^\circ} = \frac{\frac{8j \times 5e^{40^\circ}}{8j + 12 + 6j + \frac{4j}{3} + \frac{2}{3}}}{\frac{38j}{3} + \frac{38j}{3} \cdot e^{45^\circ}} = \frac{\frac{40 \cdot e^{45^\circ}}{38 \cdot \sqrt{2}}}{\frac{30 \cdot e^{45^\circ}}{19 \cdot \sqrt{2}}} \]

\[ = \frac{30}{19} \cdot e^{85^\circ} \]
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