Solution

P1, M2, L2 & 3

a) Before collision between bullet and wood : $p_i = m_B \cdot V_B$

After collision between bullet and wood : $p_f = (m_B + m_W) \cdot V_1$

Conservation of linear momentum :

$$p_i = p_f \rightarrow \boxed{V_1 = \frac{m_B}{(m_B + m_W)} \cdot V_B}$$

Velocity of wood after the bullet hits it.

b) After collision between bullet and wod : $E_i = \frac{1}{2}(m_B + m_W) \cdot V_1^2$

When the system reaches maximum height : $E_f = (m_B + m_W) \cdot g \cdot \Delta h$

$(v=0)$

Conservation of energy : $E_i = E_f \rightarrow \Delta h = \frac{V_1^2}{2g}$

$$\rightarrow \boxed{\Delta h = \left(\frac{m_B}{(m_B + m_W)}\right)^2 \cdot \frac{V_B^2}{2g}}$$

Maximum change in height of the wood block

d) Yes, because it is an inelastic collision

Before collision between bullet and wood block : $E_1 = \frac{1}{2} m_B \cdot V_B^2$

After collision between bullet and wood block : $E_2 = \frac{1}{2}(m_B + m_W) \cdot V_1^2$

$$\Delta E = E_1 - E_2 = \frac{1}{2} m_B \cdot V_B^2 - \frac{1}{2}(m_B + m_W) \cdot \frac{m_B^2}{(m_B + m_W)^2} \cdot V_B^2$$

$$\Delta E = \frac{1}{2} m_B \cdot V_B^2 \left[1 - \frac{m_B}{m_B + m_W}\right] = \boxed{\frac{1}{2} \frac{m_B \cdot m_W}{m_B + m_W} V_B^2}$$

Energy lost during collision

2nd Midterm, Adrian Lee, 2012, Correction exercice 2.

Collisions:

1) Collision m_1 / m_2 : elastic collision ⇒ we can apply the conservation of energy and the momentum (no external forces)

$$\begin{cases} \text{conservation of energy: } \frac{1}{2} m_1 V_{m_1 0}^2 + \frac{1}{2} m_2 V_{m_2 0}^2 = \frac{1}{2} m_1 V_{m_1 f}^2 + \frac{1}{2} m_2 V_{m_2 f}^2 \\ \text{momentum: } m_1 V_{m_1 0} + m_2 V_{m_2 0} = m_1 V_{m_1 f} + m_2 V_{m_2 f} \end{cases}$$

$m_1 = M$

$m_2 = \frac{M}{2}$

$V_{m_1 0} = V_0$

$V_{m_2 0} = 0$

$$\Rightarrow \begin{cases} \frac{1}{\cancel{2}} \cancel{M} V_0^2 = \frac{1}{\cancel{2}} \cancel{M} V_{1f}^2 + \frac{1}{\cancel{2}} \frac{\cancel{M}}{2} V_{2f}^2 \\ \cancel{M} V_0 = \cancel{M} V_{1f} + \frac{\cancel{M}}{2} V_{2f} \end{cases} \Rightarrow \begin{cases} V_0^2 = V_{1f}^2 + \frac{1}{2} V_{2f}^2 \\ V_0 = V_{1f} + \frac{1}{2} V_{2f} \end{cases}$$

$$\begin{cases} V_0^2 - V_{1f}^2 = \frac{1}{2} V_{2f}^2 \\ V_0 - V_{1f} = \frac{1}{2} V_{2f} \end{cases} \Rightarrow \begin{cases} (V_0 - V_{1f})(V_0 + V_{1f}) = \frac{1}{2} V_{2f}^2 \quad (1) \\ V_0 - V_{1f} = \frac{1}{2} V_{2f} \quad (2) \end{cases}$$

$$\frac{(1)}{(2)} \Rightarrow V_0 + V_{1f} = V_{2f}$$

we plug that into equation (1) ⇒ $\boxed{v_{1f} = \frac{v_{10}}{3}}$ $\boxed{v_{2f} = \frac{4}{3} v_{10}}$

2) Collision m_2 / m_3 : inelastic collision, $V_{2f} = V_{3f} = V_f$

momentum: $m_2 \frac{4}{3} v_0 = (m_3 + m_2) v_f \Rightarrow \boxed{v_f = \frac{4}{3} \frac{m_2}{m_3 + m_2} V_0}$

$m_2 = \frac{M}{2}$, $m_3 = M$ $\Rightarrow \boxed{V_f = \frac{4}{9} V_0}$

3) m_1 collides $m_2 + m_3$ if $V_{1f} > V_f$

$V_{1f} = \frac{V_0}{3}$ and $V_f = \frac{4}{9} V_0$ $\quad \frac{3}{9} < \frac{4}{9} \Rightarrow$ $\boxed{\text{there is no other collision}}$

collision if $V_{1f} < V_f \Rightarrow \frac{4}{3} \frac{M/2}{m_3 + M/2} V_0 = \frac{4}{9} V_0 \Rightarrow$ $\boxed{\text{collision if } m_3 > \frac{3}{2} M}$

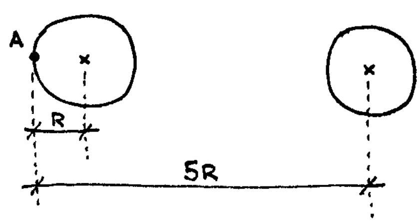
Solution

$\boxed{\text{P3, M2, L2 \& L3}}$

a) By definition: $F_g = -\dfrac{dU_g}{dr} \longrightarrow U_g = -\displaystyle\int_\infty^r F_g\,dr = -\int_\infty^r \left(-\dfrac{G\cdot mM}{r^2}\right)dr$

$$\longrightarrow U_g = -\frac{G\cdot mM}{r}\Bigg/_{\infty}^{r} = \boxed{-\frac{G\cdot mM}{r}}$$

Potential energy due to one planet and a mass m (valid if $r > R$)



At point A: $KE_A = \dfrac{1}{2} m V_{esc}^2$

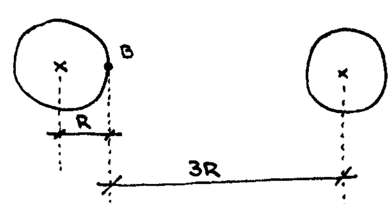
$$U_{gA} = \left(-\frac{GmM}{R}\right) + \left(-\frac{GmM}{5R}\right) = -\frac{6}{5}\,\frac{GMm}{R}$$

The minimum Vesc is given by the condition that at far away ($r \to \infty$) distance the velocity of the projectile be zero

At very far away distance ($r \to \infty$): $KE_B = 0$; $U_{gB} = 0$

Conservation of Energy: $E_A = E_B \longrightarrow \boxed{V_{esc} = \sqrt{\dfrac{12\,GM}{5R}}}$

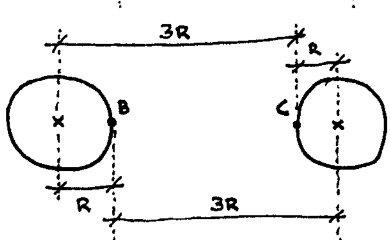
b)



Same as part (a) except: $U_{gA} = \left(-\dfrac{GMm}{R}\right) + \left(-\dfrac{GmM}{3R}\right)$

$$\longrightarrow \boxed{V_{esc} = \sqrt{\frac{8\,GM}{3R}}}$$

c)

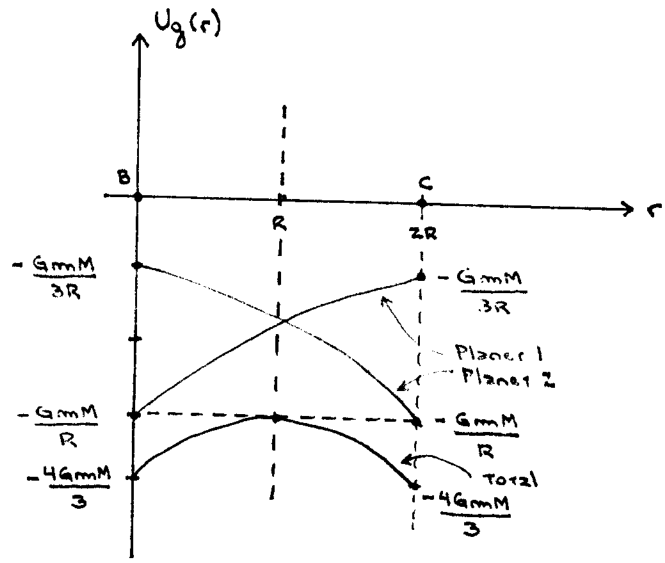


$U_{gB} = U_{gC} \longrightarrow$ It doesn't work to solve the problem like this.

Lets express the potential energy at any distance r from point B:

$$U_g(r) = \left(-\frac{GmM}{R+r}\right) + \left(-\frac{GmM}{3R-r}\right)$$

$$\longrightarrow U_g(r) = -\frac{4R\cdot GmM}{(R+r)(3R-r)}$$



Max $U_g = -\frac{GmM}{R}$ at $r=R$ (midpoint) $\longrightarrow$ The projectile just needs to make it past the midpoint between B and C.

<u>At point B</u>: $KE_B = \frac{1}{2} m V_{esc}^2$; $U_{gB} = \left(-\frac{GmM}{R}\right) + \left(-\frac{GmM}{3R}\right) = -\frac{4}{3}\frac{GmM}{R}$

<u>At midpoint</u>: $KE = 0$ (minimum V_{esc}) ; $U_g = -\frac{GmM}{R}$

Conservation of Energy: $E_B = E_{1/2} \longrightarrow \boxed{V_{esc} = \sqrt{\frac{2GM}{3R}}}$

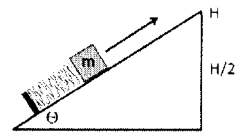
4 Block Launcher [25 pts. total]

Consider a block launched from a wedge of height H and angle θ by a spring with spring constant k. The spring is compressed from its equilibrium by a distance d. A block with mass m is placed on the compressed spring half way on the incline of the wedge with a height $H/2$

a) What is the velocity of the block as it leaves the hill? Include the effect of friction with coefficient of kinetic friction μ. Assume the spring is strong enough to launch the block from the wedge. [10 pts.]

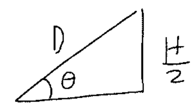
b) What is the velocity of the block when it falls back to a height $H/2$? [5 pts.]

c) If at that point in part (b) the block breaks into two pieces with mass 1/3 m and 2/3 m such that the 1/3 m piece falls straight down with the same vertical velocity as before, how far apart will the two pieces be when they land? For the answer to this section, you do not have to plug in your values from parts (a) and (b). [10 pts.]



a) $E_{initial} = \frac{1}{2}kd^2 + mg\frac{H}{2}$

$$E_{\text{top of hill}} = mgH + \frac{1}{2}mv^2 = \frac{1}{2}kd^2 + mg\frac{H}{2} - mg\cos\theta\cdot D\mu$$



$$= \frac{1}{2}kd^2 + \frac{H}{2}mg\left(1 - \frac{\cos\theta}{\sin\theta}\mu\right)$$

$\sin\theta = \frac{H/2}{D}$

$D = \frac{H}{2\sin\theta}$

No friction −3

Magnitude only −2

$$mg\frac{H}{2} + \frac{1}{2}mv^2 = \frac{1}{2}kd^2 - \frac{mgH}{2}\cot\theta\mu$$

$$\boxed{\vec{V} = \left[\frac{K}{m}d^2 - gH\cot\theta\mu - gH\right]^{1/2}\langle\cos\theta, \sin\theta\rangle}$$

b) $\cancel{mg\frac{H}{2}} + \frac{1}{2}mV_1^2 = \cancel{mgH} + \frac{1}{2}\left(Kd^2 - mgH\cot\theta\mu - \cancel{mgH}\right]$

$$V_1 = \left[\frac{K}{m}d^2 - gH\cot\theta\mu\right]^{1/2}$$

But the change in magnitude comes entirely from the vertical component of velocity:

$$\frac{K}{m}d^2 - gH\cot\theta\mu = \left(\frac{K}{m}d^2 - gH\cot\theta\mu - gH\right)\cos^2\theta + V_y^2$$

$$V_y^2 = \left(\frac{K}{m}d^2 - gH\cot\theta\mu\right)(1 - \cos^2\theta) + gH\cos^2\theta$$

$$\boxed{\vec{V}_1 = \left\langle\left(\frac{K}{m}d^2 - gH\cot\theta\mu - gH\right)^{1/2}\cos\theta, -\sqrt{\left(\frac{K}{m}d^2 - gH\cot\theta\mu\right)\sin^2\theta + gH\cos^2\theta}\right\rangle}$$

negative

c) $\vec{P}_i = m\vec{V}_i$

$$\vec{P}_f = \frac{1}{3}mV_{iy}\hat{y} + \frac{2}{3}m\left(V_x'\hat{x} + V_{iy}\hat{y}\right)$$

$$mV_{i,x} = \frac{2}{3}mV_x' \Rightarrow V_x' = \frac{3}{2}V_{i,x}$$

Time to fall: $0 = \frac{H}{2} + V_{i,y} t - \frac{1}{2} g t^2$

$$t = \frac{-V_{i,y} \pm \sqrt{V_{i,y}^2 + 4 \frac{1}{2} g \frac{H}{2}}}{-g} = \frac{V_{i,y}}{g} \mp \frac{\sqrt{V_{i,y}^2 + gH}}{g}$$

V_y is a negative number, so to get $t > 0$ we take

$$t = \frac{V_{i,y}}{g} + \frac{\sqrt{V_{i,y}^2 + gH}}{g} > 0$$

Then the blocks will be separated by

$$\boxed{D = \frac{3}{2} V_{ix} \frac{1}{g} \left(V_{i,y} + \sqrt{V_{i,y}^2 + gH} \right)}$$