Math 54, Spring 2012, F.Rezakhanlou

Each question should be answered directly. Use the back of these sheets if necessary. Justify your assertions; include detailed explanation, and show your work. No aid (including calculators) are allowed.

Your Name:

Your GSI's Name:

Your Section:

• **1.** (a)(*20 pts*) Find a matrix P that orthogonally diagonalize the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

(b) (*10 pts*) Find A^4 and find a matrix B such that $B^3 = A$. (Express your answers in terms of P.)

- **2.** (15 points) Find a basis for the column space of

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & -1 & 5 & 10 \\ 0 & -3 & 15 & 18 \\ 0 & -2 & 10 & 8 \\ 0 & 1 & -5 & -10 \end{bmatrix}.$$

What is the rank of A?

- **3.** (10 points) Given a nonzero $n \times n$ matrix A, define $< \mathbf{a}, \mathbf{b} >= (A\mathbf{a}) \cdot (A\mathbf{b})$. Under what conditions on A, $< \mathbf{a}, \mathbf{b} >$ defines an inner product for $\mathbb{R}^n$? Explain your answer.

(b) (5 points) Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors in $\mathbb{R}^n$ such that $\mathbf{a} \cdot \mathbf{b} \neq 0$. Define $T(\mathbf{v}) = \mathbf{v} - \frac{\mathbf{v} \cdot \mathbf{a}}{\mathbf{b} \cdot \mathbf{a}} \mathbf{b}$. Find the dimension of the null space and the range of T.

- **4. (True - False)** *(20 points)*

For each of the questions below, indicate if the statement is **true** or **false.** If true, **justify** (give a brief explanation or quote a relevant theorem from the course), and if false, give a counter-example or explain.

(a) If $AA^T = A^T A$, then $\|Ax\| = \|A^T x\|$.

(b) For all a, b, c, θ

$$(a\cos\theta + b\sin\theta + c)^2 \leq 2(a^2 + b^2 + c^2).$$

(c) If A and B are orthogonal matrices of the same size, then AB is an orthogonal matrix.

(d) If λ_0 is an eigenvalue of a matrix A, then the multiplicity of λ_0 as a root of the characteristic polynomial, is the same as the dimension of the eigenspace corresponding to λ_0.

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