Math 54, Spring 2011, F.Rezakhanlou

Each question should be answered directly. Use the back of these sheets if necessary. Justify your assertions; include detailed explanation, and show your work. No aid (including calculators) are allowed.

Your Name:

Your GSI's Name:

Your Section:

• **1.** (22 points) Let

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ a & 1 & 1 & 3 \\ 0 & 2 & 1 & 0 \end{bmatrix}.$$

Find the null space and column space of A when $a = 3$. For what $\mathbf{b}$, the equation $A\mathbf{x} = \mathbf{b}$ has a solution for $\mathbf{x}$?

• **2.** (20 points) Let A be as in Problem 1. Show that A is invertible when $a = 1$ and find its inverse.

• **3.** (18 points) Let A be as in Problem 1 and assume that $a \neq 3$. If $A\mathbf{x} = \mathbf{b}$ with

$$\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix},$$

use Cramer's rule to find x_2.

- **4. (True - False)** *(20 points)*
For each of the questions below, indicate if the statement is **true** or **false**. If true, **justify** (give a brief explanation or quote a relevant theorem from the course), and if false, give a counter-example or explain.

(a) If A is a square matrix such that A^2 is the zero matrix, then A is the zero matrix.

(b) If A is a square matrix, then $\det(AA^T) \geq 0$.

(c) If A is a matrix of size $m \times n$ and there exists a matrix B of size $n \times m$ such that $AB = I$, then $A\mathbf{x} = \mathbf{b}$ can be solved for $\mathbf{x}$ for every $\mathbf{b}$ in $\mathbb{R}^m$.

(d) The set of polynomials $p(t) = a_0 + a_1 t + a_2 t^2$ such that $a_1 = a_2^2$ is a subspace.

(e) Suppose that A is a matrix and $A\mathbf{x}_1 = \mathbf{b}_1$, $A\mathbf{x}_2 = \mathbf{b}_2$, $A\mathbf{x}_3 = \mathbf{b}_3$. If $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$ are linearly independent, then $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$ are linearly independent.