SOLUTION

Stat 25 --- Fall 2003
Midterm 2
Open book --- 50 minutes

Show all work to salvage points. Frame your answers. Show 3 decimal places in your answers. It is not necessary to use the continuity correction, or to do interpolations in the normal table.

Problem 1. *(25 minutes, 25 points)*
Assume that roadway cracks appear in a section of a Interstate 80 according to a Poisson process with 2 cracks per mile on average. 90% of the cracks are small, 10% of the cracks are large ones that require immediate repair. Consider a 3-mile stretch of the highway. In the following questions, you may choose one approach to answer.
A. What is the chance of at most 3 cracks?
B. What is the chance of exactly 3 cracks, at most one of which will require immediate repair?
C. I enter Interstate 80 at Berkeley and I drive East. What is the chance of finding at most 2 large cracks in the next 30 miles?

A. Average number of cracks over 3 miles : $2 \times 3 = 6$ cracks

X_1 = # cracks over 3 miles , $X_1 \sim$ Poisson(6)

$P(X \le 3) = e^{-6} + 6e^{-6} + \frac{6^2}{2}e^{-6} + \frac{6^3}{3!}e^{-6} = \boxed{0.151}$

B. X_2 = # small cracks over 3 miles $X_2 \sim$ Poisson (6×0.9)

X_3 = # large cracks over 3 miles $X_3 \sim$ Poisson (6×0.1)

I must have

either : 3 small and 0 large

$P(X_2 = 3) P(X_3 = 0) = \frac{(5.4)^3 e^{-5.4}}{3!} \times e^{-0.6} = 0.065$

or : 2 small and 1 large

$P(X_2 = 2) P(X_3 = 1) = \frac{(5.4)^2 e^{-5.4}}{2!} \frac{(0.6) e^{-0.6}}{1} = 0.217$

Answer : $0.065 + 0.217 = \boxed{0.867}$

C. Average number of large cracks over 30 miles : $2 \times 30 \times 0.1 = 6$

X_4 = # large cracks over 3 miles $X_4 \sim$ Poisson(6)

$P(X_4 \le 2) = e^{-6} + 6e^{-6} + \frac{6^2}{2}e^{-6} = \boxed{0.062}$

SOLUTION

Problem 2. *(25 minutes, 25 points)*
A large a airplane has 100 first class seats, and 400 economy class seats. Weights of first class passengers (with baggage) are assumed to be normally distributed with mean 260 pounds and SD 40 pounds. Weights of economy class passenger (with baggage) are assumed to be normally distributed with mean 180 pounds and SD 30 pounds. If all passengers with their baggage weigh more than 50 tons (100,000 pounds), the airplane is considered to be overloaded and will not be authorized to depart. It is known that 5% and 8% of the people having reservations for respectively first class and economy class fail to show up.
A. Suppose tomorrow's flight is full, that is, all seats are occupied. Call T the total weight of all passengers and their luggage. What is the distribution of T? What is the chance of overload?
B. For tomorrow's flight, the airline has taken 101 revervations for first class and 420 reservations for economy class. Call X1, X2 the numbers of people who will show up for first class and economy class respectively. What are the distributions of X2 and X2? What is the chance that nobody will be bumped off the flight?

A. $n = 100$ $X_1 \dots X_n$ are weights of 1st class passengers

$X_i \sim N(260, 40^2)$, $X_1 + \dots + X_n \sim N(100 \times 260, 100 \times 40^2)$

$m = 400$ $Y_1 \dots Y_m$ are weights of economy class passengers

$Y_i \sim N(180, 30^2)$ $Y_1 + \dots + Y_m \sim N(400 \times 180, 400 \times 30^2)$

$T = X_1 + \dots + X_n + Y_1 + \dots + Y_m \sim N(100 \times 260 + 400 \times 180, 100 \times 40^2 + 400 \times 30^2)$

$\boxed{T \sim N(98000, 520000)}$

$P(\text{Overload}) = P(T > 100\,000) = P\left(Z > \frac{100\,000 - 98000}{\sqrt{520000}}\right)$

$= P(Z > 2.77) = 1 - 0.9972 = \boxed{0.0028}$

B. $\boxed{X_1 \sim bin(101, 0.95)}$ $\boxed{X_2 \sim bin(420, 0.92)}$

$P(X_1 \leq 100) = P\left(\frac{X_1 - n_1p_1}{\sqrt{n_1p_1q_1}} \leq \frac{100 - 101 \times 0.95}{\sqrt{101 \times 0.95 \times 0.05}}\right) \approx P\left(Z \leq \frac{4.05}{2.19}\right)$

$= P(Z \leq 1.85) = 0.9678$

$P(X_2 \leq 400) = P\left(\frac{X_2 - n_2p_2}{\sqrt{n_2p_2q_2}} \leq \frac{400 - 420 \times 0.92}{\sqrt{420 \times 0.92 \times 0.08}}\right) \approx P\left(Z \leq \frac{13.6}{5.560}\right)$

$= P(Z \leq 2.44) = 0.9927$ $\qquad 0.9678 \times 0.9927 = \boxed{0.96}$