UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME132 Dynamic Systems and Feedback

Midterm II **Spring 2010**

Closed Book and Closed Notes. One 8.5 × 11 sheet (only front) of handwritten notes allowed. Scientific calculator without graphics allowed.

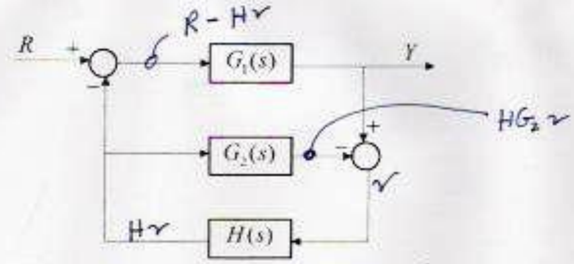
Your Name:

Please answer all questions.

Problem:	1	2	3	4	Total
Max. Grade:	20 pts	40 pts	25 pts	15 pts	100
Grade:					

1. Assume $G_1(s)$, $G_2(s)$, and $H(s)$ are transfer functions of linear systems.

   (a) Compute the transfer function from R to Y in the figure below



$$r = Y - HG_2 r \Rightarrow r = \frac{Y}{1+HG_2}$$

$$Y = G_1 R - G_1 H r$$

$$Y = G_1 R - \frac{G_1 H Y}{1+HG_2}$$

$$Y = \frac{G_1}{1+\frac{G_1 H}{1+HG_2}} R \quad \text{or} \quad \boxed{Y = \frac{(1+HG_2)G_1}{1+HG_2+HG_1} R}$$

(b) Suppose that the transfer functions are given as

$$G_1(s) = \frac{1}{s+1},\ G_2(s) = \frac{1}{s+2},\ H(s) = \frac{s+2}{s+5}.$$

Express the transfer function from R to Y in terms of the variable s.

$$Y = \frac{\left[1 + \left(\frac{s+2}{s+5}\right)\left(\frac{1}{s+2}\right)\right]\frac{1}{s+1}}{1 + \frac{s+2}{s+5}\cdot\frac{1}{s+2} + \frac{s+2}{s+5}\cdot\frac{1}{s+1}} R$$

$$Y = \frac{\frac{1}{s+1} + \frac{1}{(s+5)(s+1)}}{1 + \frac{1}{s+5} + \frac{s+2}{(s+5)(s+1)}} R = \frac{s+5+1}{(s+5)(s+1)+s+1+s+2} R$$

$$Y = \frac{s+6}{s^2+8s+8} R$$

(c) What is the characteristic equation associated to the differential equation of the closed-loop system?

characteristic equation is . $s^2+8s+8=0$

2. A process, with input u, disturbance d, and output y is governed by

$$\dot{y}(t) = y(t) + u(t) + d(t)$$

where $u(t)$ is the input, $d(t)$ is the disturbance, and $y(t)$ is the output. The initial condition is $y(0) = 0$.

(a) Is the process stable?

$$\dot{y} - y = u + d$$

$$-1 < 0 \Rightarrow \text{unstable}$$

(b) A PID (Proportional Integral Derivative) controller is proposed

$$u(t) = K_D\dot{r}(t) + K_P[r(t) - y(t)] + K_I z(t)$$
$$\dot{z}(t) = r(t) - y(t)$$

with $z(0) = 0$. Eliminate z and u, and determine the closed-loop differential equation relating the variables (y, r, d).

$$\ddot{y} = \dot{y} + \dot{u} + \dot{d}$$

$$\ddot{y} = \dot{y} + K_D\ddot{r} + K_P[\dot{r} - \dot{y}] + K_I(r - y) + \dot{d}$$

$$\boxed{\ddot{y} + [K_P - 1]\dot{y} + K_I y = K_D\ddot{r} + K_P\dot{r} + K_I r + \dot{d}}$$

(c) For what values of K_D, K_P and K_I is the closed-loop system stable?

$$K_P - 1 > 0 \Rightarrow \boxed{K_P > 1}$$

$$\boxed{K_I > 0}$$

$$\boxed{K_D \in R}$$

(d) The closed-loop system is of 2nd order. Assume $y(0^-) = 0$ and $\dot{y}(0^-) = 0$. Find appropriate values of K_P, K_I and K_D so that the closed-loop system characteristic polynomial has:

i. Complex roots described by $\zeta \geq 0.8$, $\omega_n = 0.5$.

ii. When $r(t)$ is a step input and $d = 0$, then 1) $y(0^+) = 0$, and 2) $\dot{y}(0^+) \geq 1.9$.

i. $w_n = 0.5 \Rightarrow \boxed{K_I = w_n^2 = 0.25}$

Complex roots $\Rightarrow \zeta < 1$ (Note that $\zeta = \frac{K_p - 1}{2w_n} = \frac{K_p - 1}{2(0.5)} = K_p - 1$)

$K_p - 1 < 1 \Rightarrow \boxed{K_p < 2}$

$\zeta \geq 0.8 \Rightarrow K_p - 1 \geq 0.8 \Rightarrow \boxed{K_p \geq 1.8}$

ii. $d = 0 \Rightarrow$ the closed loop ODE is

$$\ddot{y} + [K_p - 1]\dot{y} + K_I y = K_D \ddot{r} + K_p \dot{r} + K_I r + 0$$

$a_1 = K_p - 1$; $b_0 = K_D$; $b_1 = K_p$

$y(0^+) = 0 + K_D = 0 \Rightarrow \boxed{K_D = 0}$

$\dot{y}(0^+) = 0 + K_p - 0 \geq 1.9 \Rightarrow \boxed{K_p \geq 1.9}$

Final answers $\boxed{K_D = 0}$; $\boxed{K_I = 0.25}$; K_p can be anything in $[1.9, 2[$

I will choose $\boxed{K_p = 1.9}$

(e) For the K_P, K_I and K_D values you found, sketch the response of y due to a unit-step disturbance d, assuming r is identically zero, and assuming all initial conditions are zero. (If you didn't successfully solve part (d), then use $\zeta = 0.8$, $\omega_n = 0.5$ and $y(0^-) = 0$, $\dot{y}(0^-) = 0$).

~~[illegible]~~

$\ddot{y} + 0.9\dot{y} + 0.25y = \dot{d}$

Most of the grade was given for calculating the following values.

~~[illegible]~~

We have $w_n = 0.5$, $\zeta = \frac{0.9}{(2)(0.5)} = 0.9$

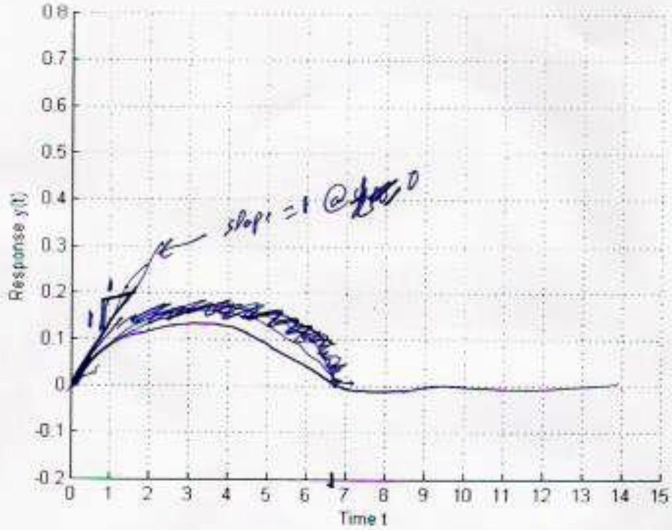
(1) Settling time $= \frac{3}{\zeta w_n} = 6.667$

(2) number of oscillations before steady state $\approx \frac{3\sqrt{1-\zeta^2}}{2\pi\zeta} = 0.23$

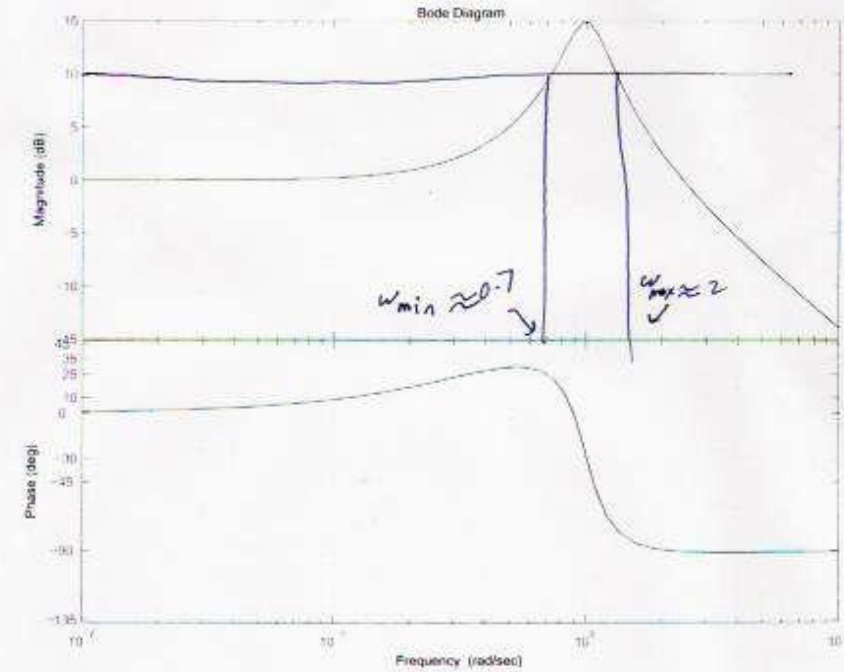
(3) $y(0^+) = 0$

$\dot{y}(0^+) = 1$

(4) $y_{ss} = 0$ [because $\dot{d} = 0$ and $r = 0$]


slope = 1 @ t = 0
Response y(t)
Time t

4. [Note: In this question, approximate anwers are expected.] Consider the SLODE $\ddot{y} + 0.4\dot{y} + y = 2\dot{u} + u$, where $u(t)$ is a sinusiodal input, i.e., $u(t) = A sin(\omega t)$, and all initial conditions are zero. The figure below shows the bode diagram of the SLODE.



(a) What is the frequency range $[\omega_{min}, \omega_{max}]$ such that the input $u(t)$ leads to a steady-state sinusoidal output $y(t)$ amplified at least 3.16 times with respect to the input ? (In math terms, find ω_{min} and ω_{max}, such that $y_{ss} = M sin(\omega t + \varphi)$, where $M \geq 3.16A$, for all $\omega \in [\omega_{min}, \omega_{max}]$).

$20 \log 3.16 \approx 10$

$w \in [0.9, 2]$

(b) Compute the output $y(t)$ for the input $u(t) = 3sin(0.1t) + 5sin(0.5t) + 2sin(t)$

$\boxed{\text{Approximate answers are expected}}$

$|G|_{db} @ w=0.1 \approx 0db \qquad \varphi @ w=0.1 \approx 10^\circ$

$|G|_{db} @ w=0.5 \approx 7db \qquad \varphi @ w=0.5 \approx 26^\circ$

$|G|_{db} @ w=1 \approx 15db \qquad \varphi @ w=1 \approx -35^\circ$

Convert

$$|G|_{db} = 20 \log|G| \Rightarrow |G| = 10^{\left(\frac{|G|_{db}}{20}\right)}$$

$$\varphi \text{ in degrees} = \varphi^{degree} = \varphi^{radians} \times \frac{180}{\pi}$$

$$\Rightarrow \varphi^{radians} = \varphi^{degree} \times \frac{\pi}{180}$$

$|G|_{@w=0.1} = 1 \qquad \varphi^{rad}_{@w=0.1} = 0.17$

$|G|_{@w=0.5} = 2.2 \qquad \varphi^{rad}_{@w=0.5} = 0.45$

$|G|_{@w=1} = 5.6 \qquad \varphi^{rad}_{@w=1} = -0.61$

$$y(t) = |G|_{@w=0.1} \times 3\sin(0.1t + \varphi^{rad}_{@w=0.1}) + |G|_{@w=0.5} \times 5\sin(0.5t + \varphi^{rad}_{@w=0.5}) + |G|_{@w=1} \times 2\sin(t + \varphi^{rad}_{@w=1})$$

$$\Rightarrow \boxed{y(t) = 3\sin(0.1t + 0.17) + 2.2 \times 5\sin(0.5t + 0.45) + 5.6 \times 2\sin(t - 0.61)}$$