ME 132, Spring 2005, Quiz # 1

# 1	# 2	# 3	TOTAL
10	20	15	45

Fact: Both roots of $\lambda^2 + a_1\lambda + a_2 = 0$ have negative real-parts if and only if $a_1 > 0$ and $a_2 > 0$.

Fact: All three roots of $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$ have negative real-parts if and only if $a_1 > 0, a_3 > 0$, and $a_1 a_2 > a_3$.

1. Your answers in part 1a and 1c both should have two free constants. I would like there to be no $\sqrt{-1}$ in the answers, just exponentials, cos and sin.

   (a) What is the general form of real (as opposed to complex) solutions to the differential equation

   $$\ddot{y}(t) + 2\dot{y}(t) + 17y(t) = 0$$

   (b) The solutions in part 1a are made up of an exponentially decaying envelope, superimposed on a sinusoid. What (approximately) is the ratio

   $$\frac{\text{Period} - \text{of} - \text{Oscillation}}{\text{Time} - \text{to} - \text{Decay}}$$

   Explain.

   (c) What is the general form of real solutions to the differential equation

   $$\ddot{y}(t) + 2\dot{y}(t) + 17y(t) = -51$$

2. A process has a very simple model,

$$y(t) = Hu(t) + w(t).$$

The control input is u, the disturbance input is w and the output is y. Here, H is simply a gain, ie., the behavior of the system is "static" - it is not governed by a differential equation.

The goal of control is to make the process output y follow a reference input r, even in the presence of nonzero disturbances w, <u>and</u> slight unknown variations in H. In order to achieve this, we use an integral controller

$$\begin{aligned} \dot{z}(t) &= r(t) - y(t) \\ u(t) &= K_I z(t) \end{aligned}$$

Here, r is the reference input.

(a) Combine the the process model, and the controller equations to (eliminating z and u) get a relationship between the process output y, the two "forcing" functions r and w, and any of their derivatives.

(b) Under what conditions (on H and K_I) is the closed-loop system stable?

(c) Assume that K_I is chosen so that the closed-loop system is stable. Does a 20% change in H (ie., H changing to $0.8H$ or $1.2H$) affect stability of the closed-loop system?

(d) Assume that K_I is chosen so that the closed-loop system is stable. If $r(t) \equiv \bar{r}$ and $w(t) \equiv \bar{w}$ for all $t \geq 0$ ($\bar{r}$ and $\bar{w}$ are some fixed constant values), what are the steady-state values (in terms of $\bar{r}, \bar{w}, K_I, H$) of y and u, defined as

$$\lim_{t\to\infty} y(t) \qquad\qquad \lim_{t\to\infty} u(t)$$

(e) How does a 20% change in H affect (approximately) the steady-state values of y and u derived in part (2d) above?

(f) Assume that K_I is chosen so that the closed-loop system is stable. What is the time-constant of the closed-loop system? Approximately how does a 20% change in H (ie., H changing to $0.8H$ or $1.2H$) affect (approximately) the time constant?

3. A closed-loop system is shown below. Here H and K are positive constants, namely $H = 4$, and $K = 1$.

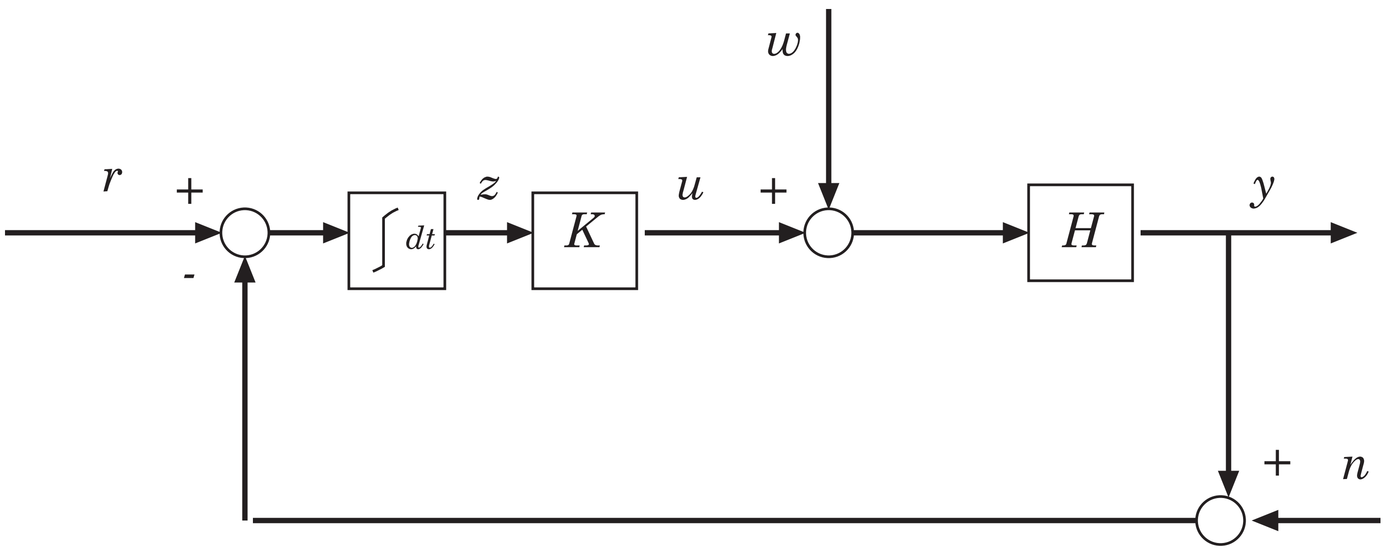
(a) By eliminating y and u from the equations, find the constants A, B_1, B_2 and B_3 such that the internal variable z is related to the external forcing functions (r, w, n) in the form

$$\dot{z}(t) = Az(t) + B_1 r(t) + B_2 w(t) + B_3 n(t)$$

(b) Express the variable y as a combination of z, r, w, n, namely find the constants C_1, D_{11}, D_{12} and D_{13} such that

$$y(t) = C_1 z(t) + D_{11} r(t) + D_{12} w(t) + D_{13} n(t)$$

Missing Fig. in Problem 3 spring2005-quiz-1

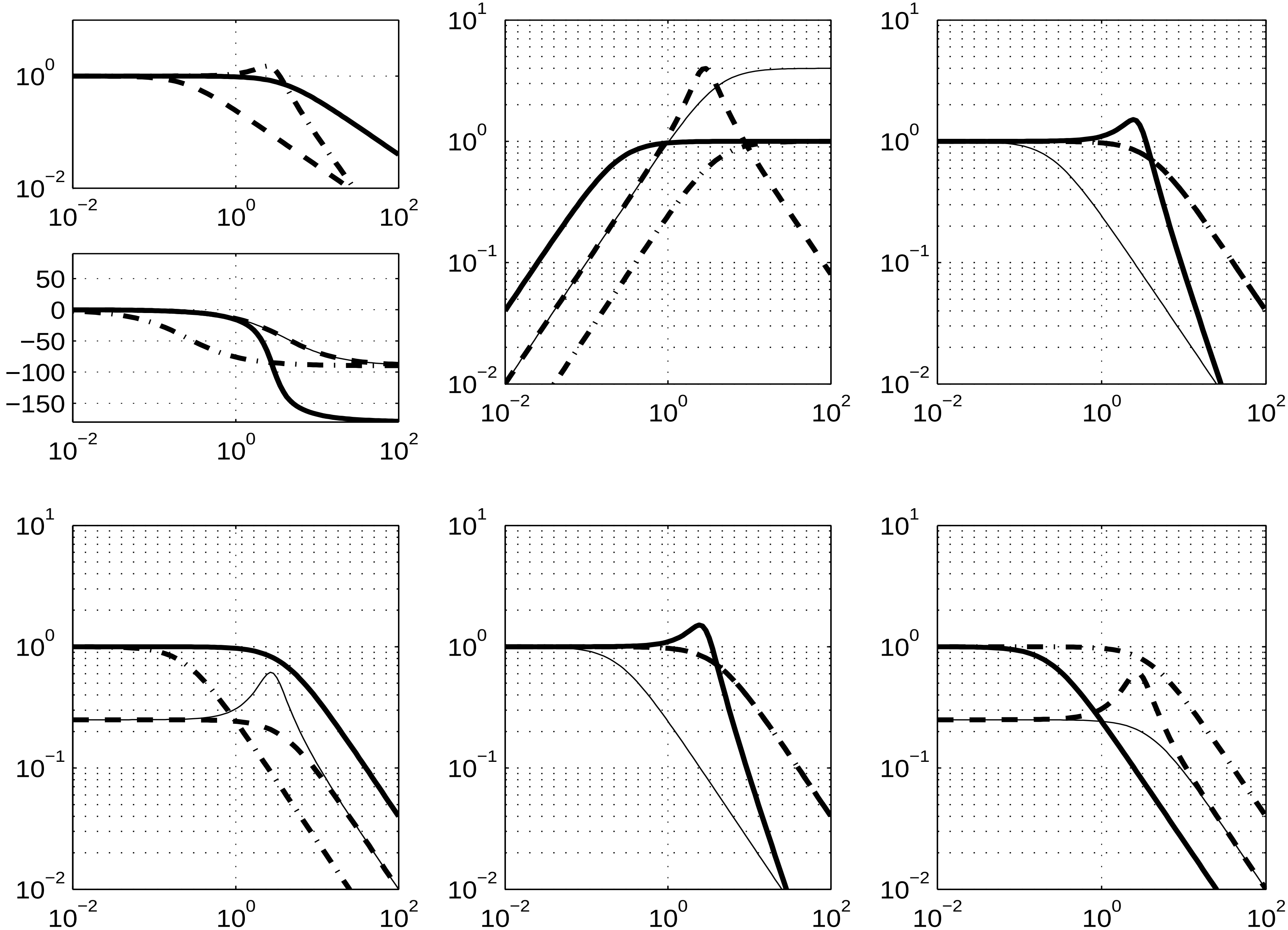


(c) Express the variable u as a combination of z, r, w, n, namely find the constants C_2, D_{21}, D_{22} and D_{23} such that

$$u(t) = C_2 z(t) + D_{21} r(t) + D_{22} w(t) + D_{23} n(t)$$

(d) Shown on the next page is the frequency-response matrix (2×3) from the three external inputs (r, w, n) to the two "outputs-of-interest" (y, u). In 5 of the cases, only the magnitude is shown. In the case of input/output pair (r, y), both magnitude and phase are shown.

In each axes, 3 or 4 lines are shown, though only one is correct. In each case, mark the correct frequency response curve.


10^0
10^-2
10^-2 10^0 10^2
50
0
−50
−100
−150
10^1
10^-1