In class, Monday, September 19, 2011

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ME 106 Mid-term Exam I

No partial credit if your answer has incorrect dimensions

The figure on the next page shows a fish tank open to the atmosphere at the top with pressure P_{atm}. The tank extends in the y direction (into the paper) from $y = -L/2$ to $y = L/2$, so the thickness in the y direction is L. There is a partition down the middle of the tank at $x = 0$ that extends from the bottom of the tank at $z = z_B$ to the top of the figure. On the left of the partition, a fluid of density ρ_3 occupies the volume with $\bar{z} \leq z \leq 0$, and a fluid of density ρ_1 occupies the volume with $z_B \leq z \leq \bar{z}$. On the right of the partition, a fluid of density ρ_4 occupies the volume with $\bar{z} \leq z \leq z_4$, and the fluid of density ρ_1 occupies the volume with $z_B \leq z \leq \bar{z}$. The partition is sealed by a hinged gate between points **1** and **2**, where the hinge is located at point **1** as shown in the figure, and the gate has a right angle at $z = \bar{z}$. The points **1** and **2** are located at $x = 0$ and along the plane of symmetry at $y = 0$. The upper section of the gate makes an angle of 30° with respect to the vertical axis; the lower section of the gate makes an angle of 60° with respect to the vertical axis.

1) Find the torque on the hinge at point **1** due to the fluids with densities ρ_3 and ρ_4.

a) Start your calculation by finding the directions x, y, and/or z (if any) in which the torques are zero. If the torque is zero in some direction, explain why, but don't waste your time working out the calculus to prove it.
b) Find the vector **r** that goes from point **1** to the arbitrary point (x, y, z) in the part of the hinged gate with $z \leq \bar{z}$.
c) Write down the normal vector that goes from the fluid with density ρ_3 to the upper section of the gate; write down the normal vector from the fluid with density ρ_4 to the upper section of the gate.
d) Write down the pressure as a function of z on the left side of the partition; write down the pressure as a function of z on the right side of the partition.
e) Find the total torque around point **1** due to the fluids with densities ρ_3 and ρ_4.

2) Find the total torque on the gate at point **2** due to the fluid with density ρ_1, which acts

on both the left and right sides of the gate.

3) In class we showed that the torque $\mathbf{\Gamma}_1$ that a fluid exerts around point **1** is related to the torque $\mathbf{\Gamma}_2$ that the same fluid exerts around point **2** by the relation $\mathbf{\Gamma}_1 = \mathbf{\Gamma}_2 + \mathbf{r}_{12} \times \mathbf{F}$, where $\mathbf{F}$ is the force on the gate, and $\mathbf{r}_{12}$ is the vector that goes from point **1** to point **2**. By using this relationship and finding $\mathbf{r}_{12}$ and only those components of $\mathbf{F}$ *that are needed to do the calculation*, find the torque around point **1** due to the fluid with density ρ_1.

4) Under some conditions the total torque at point **1** due to all three of the fluids is zero. Find a relation among ρ_1, ρ_3, ρ_4, $\bar{z}$, z_4, and z_B such that the total torque is zero. (Don't supply a trivial relation, such as stating that all of the densities are zero.)