Abstract Algebra - Midterm 2

Alexander Paulin

November 5, 2010

Question 1

Let G and H be two groups. Let $\phi : G \to H$ be a group homomorphism.

1. Define $ker(\phi)$. Prove it is a subgroup of G.
2. Prove $ker(\phi)$ is normal in G.
3. State, without proof, the First Isomorphism Theorem.
4. State, without proof, the Third Isomorphism Theorem.
5. Prove that if both G and H are finite and ϕ is surjective, then the number of subgroups of G is greater than or equal to the number of subgroups of H.

Question 2

Let R be a ring.

1. Define what it means for R to be commutative.
2. Define what it means for R to be an integral domain. Given an example of a commutative ring which is not an integral domain.

Question 3

1. State the basis theorem for finitely generated Abelian groups.
2. Let $n, p \in \mathbb{N}$ with p a prime number. Prove that, up to isomorphism, there is only one finite Abelian group of size p^n all of whose non-zero elements have order p.
3. Prove that if $\mathbb{F}$ is a finite field of characteristic p then it has order p^n for some $n \in \mathbb{N}$. You may assume any results from lectures as long as they are stated clearly.