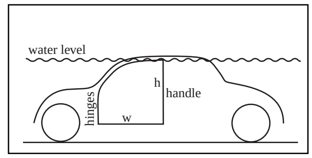
Ö. Savaş

NAME: **SOLUTIONS**

11:10-12:30, Thursday, February 26, 2004

ME 106.2 FLUID MECHANICS
EXAM 1 – open book

1.(25%) Determine the minimum force needed to open the door (1m high, 0.8m wide) of a car submerged to its roof in water. The inside pressure is atmospheric and the doors are sealed.



$$F = \underbrace{\frac{1}{2}}_{mom.\ arm} \left(\underbrace{\frac{1}{2}\rho g h}_{ave.\ pres.} \times \underbrace{h \times w}_{area} \right) = \frac{1}{4}\rho g w h^2 = \frac{1}{4} \times 1000 \times 9.8 \times 0.8 \times 1.0^2 = 1960\ N$$

2.(25%) Katabatic winds, also known as mountain breezes, form when a relatively cool wind descends from a high elevation, such as the notorious Santa Ana winds of Southern California. Let us assume that Santa Ana starts in the Rockies at an elevation of 1000 meters, at a temperature of 8.5°C, and speed of 2 m/s and descends to Los Angeles, which may be assumed to be at sea level. Assuming the wind to be adiabatic and reversible (which is certainly not true), determine the speed and the temperature of the wind in Los Angeles. You may use the standard atmospheric values when needed (Table C2).(20+5)

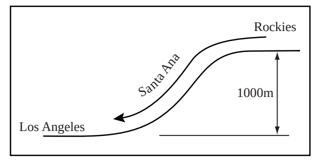
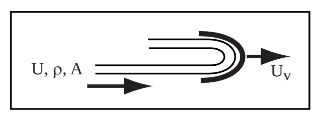


Table C.2: $T_r = 281.5K$, $p_r = 8.988 \times 10^4$ Pa, $\rho_r = 1.112$ kg/m^3, $p_{la} = p_{atm} = 1.013 \times 10^5$ Pa, $\gamma = 1.4$

$$\left[\frac{\gamma}{\gamma-1}\frac{p}{\rho} + \frac{1}{2}U^2 + gz\right]_r = \left[\frac{\gamma}{\gamma-1}\frac{p}{\rho} + \frac{1}{2}U^2\right]_{la}, \qquad \left.\frac{p}{\rho^\gamma}\right|_r = \left.\frac{p}{\rho^\gamma}\right|_{la} \Longrightarrow \left.\frac{T}{p^{\frac{\gamma-1}{\gamma}}}\right|_r = \left.\frac{T}{p^{\frac{\gamma-1}{\gamma}}}\right|_{la}$$

$$U_{la} = \sqrt{\frac{2\gamma}{\gamma-1}\frac{p_r}{\rho_r}\left(\left(\frac{p_{la}}{p_r}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right) + U_r^2 + 2gz_r} = 198\ m/s \ \ \&\ \ T_{la} = T_r \times (p_{la}/p_r)^{(\gamma-1)/\gamma} = 291K = 18°C$$

3.(25%) Consider a water jet of velocity $(U, 0)$ and cross-section A which is impinging on a moving vane. The vane turns the jet 180°. Determine the velocity U_v of the vane for maximum power extraction from the water jet. What is the maximum power. (20+5)



$$\dot{W} = \underbrace{\rho A(U - U_v)}_{mass\ flux} \times \underbrace{2(U - U_v)}_{mom.\ ch.} \times U_v = 2\rho A U_v (U - U_v)^2 \Longrightarrow d\dot{W}/dU_v = 0 \Rightarrow U_v = U/3 \Rightarrow \dot{W}_{max} = \frac{8}{27}\rho A U^3$$

4. (25%) Consider the two-dimensional velocity field

$$(u, v) = \Big(cosh(y)\ cos(x),\ -sinh(y)\ cos(x + \pi/2)\Big)$$

(a) Determine the equation of and sketch the streamline passing through $(x, y,) = (0, 0)$. (5+5)

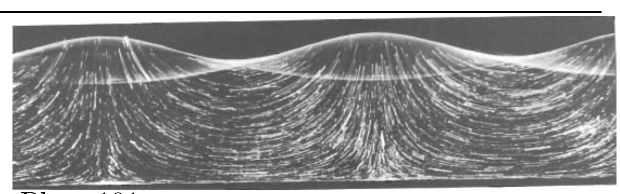
(b) Determine the equation of and sketch the streamline passing through $(x, y,) = (0, 1)$. (10+5)

Hint: For sketching purposes, you may take $sinh(y) \approx y$.

standing water waves

$$\frac{dx}{u} = \frac{dy}{v} \Rightarrow \frac{cos(x + \pi/2)}{cos(x)}dx = \frac{-sin(x)}{cos(x)}dx = \frac{-cosh(y)}{sinh(y)}dy$$

$$\Rightarrow sinh(y) \times cos(x) = const.$$



M. Van Dyke, *An Album of Fluid Motion*, Parabolic, 1982. Plate 191.