Problem 1: POINT BREAKDOWN:

(a) $\boxed{7}$ Points total

↳ −2 points for over 2m rather than over 1 m.

(b) $\boxed{13}$ Points total

5 points → solve eqns to determine that block must hit ramp

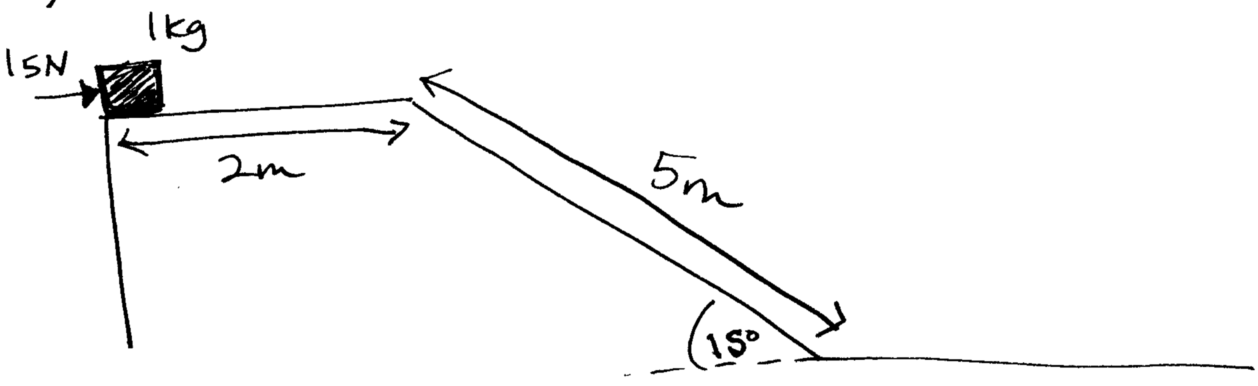
3 points → recognize once on ramp, accel is $a = \pm g\sin\theta$

2 points → recognize down the ramp you can use $v^2 = v_0^2 + 2a\Delta x$

2 points → actually calc. point on ramp where block hits, V_{fx}, V_{fy}

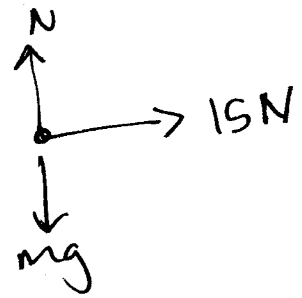
1 pt → recognize that you need to think about what happens to velocity when block hits → ramp absorbs ⊥ velocity? voice your assumption.

1)



a) How quickly is box moving when it reaches the edge of the platform?

<u>Plan:</u> determine acceleration; use kinematics eq'ns



$\Sigma F_x = 15N = ma$; $\boxed{a = \frac{15N}{1\,kg}}$

Box experiences a over 1m

$V_f^2 = V_i^2 + 2a\Delta x$

$V_f^2 = 0 + 2(15\frac{N}{kg})(1m)$

$V_f^2 = 30$

$V_f = \sqrt{30} = \boxed{5.48\ m/s}$ (same @ end since no friction or force over last 1m)

(b) Does block make contact? If so, what is velocity @ bottom of ramp?

Plan: get eq'ns for block, eq'n for line of ramp & see where (if) they intersect.

For block:

$y = mx + b$

$$y = \frac{-5\sin\theta}{-5\cos\theta}x + 0$$

$$\boxed{\begin{aligned} y &= -\tan\theta\, x \\ y &= -.27x \end{aligned}}$$

Assuming $(0,0)$ is top of ramp,

$x(t) = v_0 t$; $y(t) = \frac{-gt^2}{2}$

so $\frac{y}{x} = -\tan\theta \Rightarrow \frac{\frac{1}{2}gt}{v_0} = \tan\theta$

so $\boxed{t = .3s}$ when block hits ramp

· Plug back in for v_x, v_y :

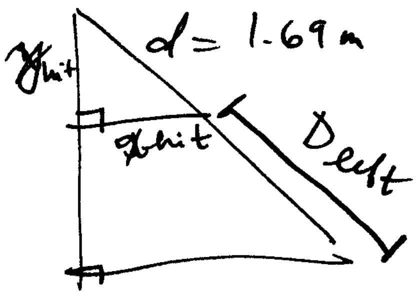
$v_{x,hit} = v_0$ (always) $= 5.48$ m/s

$v_{y,hit} = -gt = -2.94$ m/s

· what are x, y values when it hits?

$x_{hit} = 1.635$ m, $y_{hit} = .441$ m (from $y(t)$, $x(t)$)

so

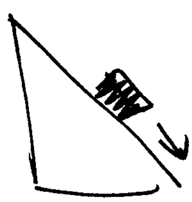


$d = \sqrt{x_{hit}^2 + y_{hit}^2}$

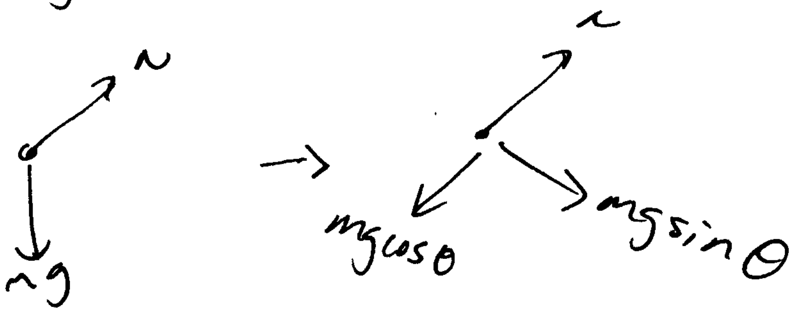
$= 1.69$ m

$D_{left} = 5\text{m} - 1.69\text{m} = \boxed{3.31 \text{ m}}$

When it is sliding down block,



FBD:



$\cancel{m}a = \cancel{m}g\sin\theta$

$\boxed{a = g\sin\theta \text{ along ramp}}$

Now, the annoying part:

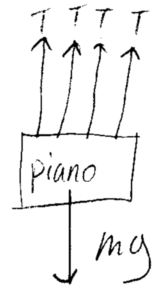
Since the direction of the final velocity is not along the ramp, we need to consider what happens during the collision — most likely, the ramp will absorb the velocity $\perp$ to it & allow the block to continue down w/ initial velocity parallel. Since you aren't expected to know about an interaction like this, if you calculated the magnitude of the total velocity,

$$V_f = \sqrt{v_{f,x}^2 + v_{fy}^2} = 6.12 \text{ m/s}$$

& then assumed this velocity was where you started$_x$ on the ramp that also received full credit.

Using $V_f^2 = V_i^2 - 2a\Delta x$ where $\Delta x = 3.31$, $a = g\sin\theta$, $v_i = 6.12$ m/s

2.) a.) (8 pts)



The force you apply to the rope is the tension on the ropes, and each rope pulls with a force of tension equal to the tension you create.

$F_{net} = ma$

The minimum force needed to lift the piano would be when the piano is just about to move and $a=0$.

$$4T - mg = 0 \quad , F=T$$

$$4F - mg = 0$$

$$4F = mg$$

$$F = \frac{mg}{4} = \frac{(160\ kg)(9.8\ m/s^2)}{4} = \boxed{392\ N}$$

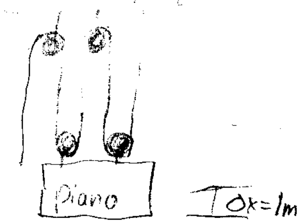
b.) The total downward force is going to be the combination of all the tensions pulling down on the ceiling. This is just $4T$ plus the downward component of the force you apply at an angle Θ. (8 pts)

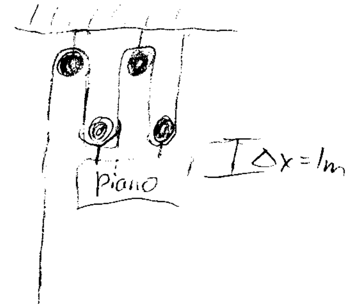
$$F_{down} = 4T + T\cos\Theta$$

$$T = F$$

$$\boxed{F_{down} = 4F + F\cos\Theta}$$

c.) The amount of rope pulled down is the amount of rope of rope on the pulleys used to move the piano up 1m. (4 pts)





Each rope is contracted by 1m by pulling the piano up so the total rope pulled is $4 \cdot 1m = 4m$

$\boxed{4m}$

Problem 3

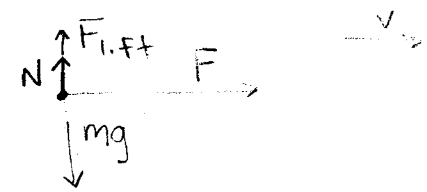
An airplane of mass 300,000 kg lifts off a runway of length $L = 3\ km$. The engines provide a forward force of F. The vertical lift of the wings is proportional to the velocity squared of the aircraft: $F_{lift} = (1000\ kg/m)v^2$

a) Draw a Free Body Diagram of the airplane (ignoring friction and air resistance). (4 points)

b) Determine the minimum force, F_{min}, required of the engines for the airplane to lift off before the edge of the runway. (8 points)

c) If $F = 1.5F_{min}$, graph the normal force on the plane versus the horizontal position of the plane, for $0 \leq x \leq L = 3\ km$. (8 points)

a)



b) If the airplane lifts off, N = 0 so

$F_{lift} = mg \longrightarrow v_{min}^2 = 2940\ m^2/s^2$

$v_{min}^2 = \cancel{[v(x=0)]^2} + 2ax \longrightarrow a_{min} = \frac{v_{min}^2}{2x} = \frac{2940\ m^2/s^2}{2(3\times10^3\ m)} = .49\ m/s^2$

$F_{min} = ma_{min} = (300{,}000\ kg)(.49\ m/s^2) = \boxed{1.47\times10^5\ N}$

c) Since Force = Mass x acceleration,

$a = 1.5a_{min} = 1.5(.49\ m/s^2) = .735\ m/s^2$

$[v(x)]^2 = v(x=0)^2 + 2ax = 2(.735\ m/s^2)x = 1.47\ m/s^2\ x$

$F_{lift}(x) = 1000\ kg/m\ [v(x)]^2 = (1000\ kg/m)(1.47\ m/s^2)\ x = (1470\ kg/s^2)\ x$

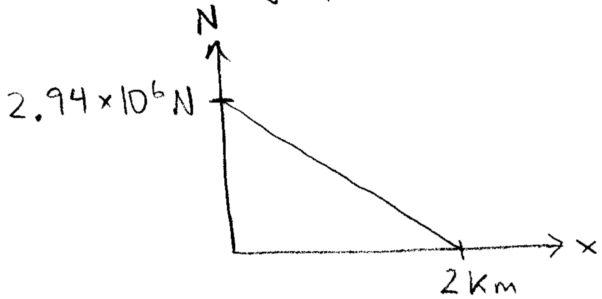
From the FBD,

$N = mg - F_{lift} = 2.94\times10^6\ N - (1470\ kg/s^2)\ x$

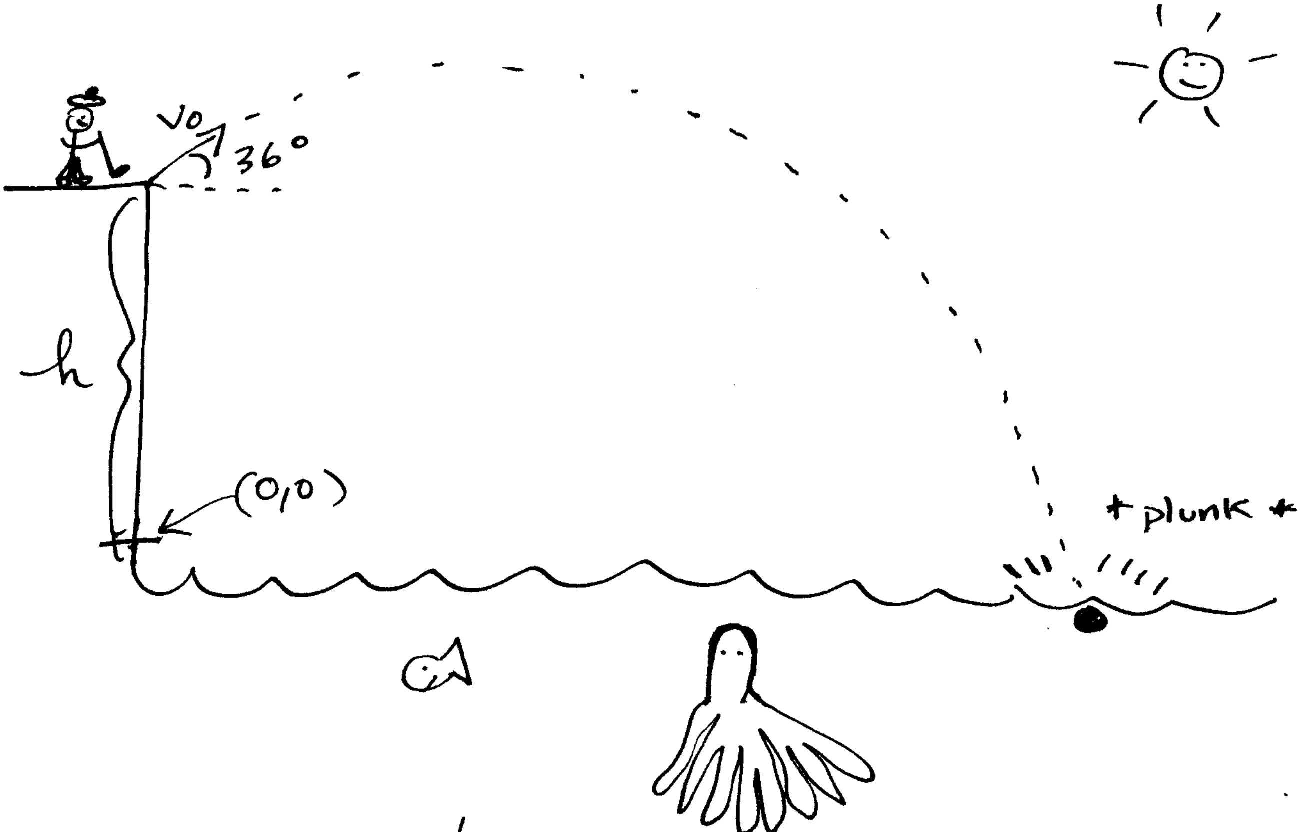
The plane lifts off when

$F_{lift} = mg \rightarrow (1470\ kg/s^2)\ x = (3\times10^5\ kg)(9.8\ m/s^2) \longrightarrow x = 2\ km$

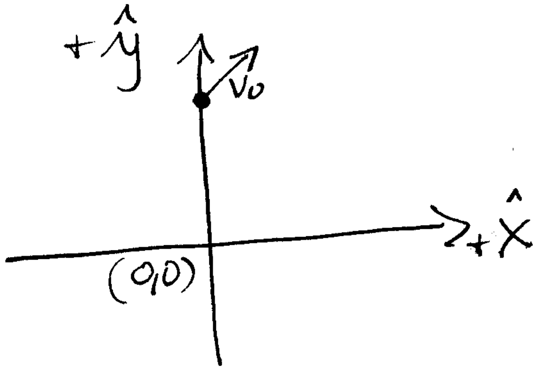
So the graph looks like:



4)



Kinematic Eq'ns!



I define y=0 at ocean and x=0 from where the ball starts.

y - dir :

$$y(t) = y_0 + v_0 t + \frac{1}{2} a t^2$$

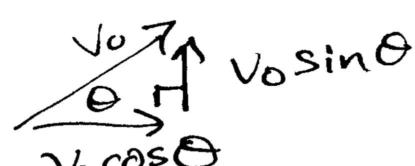
$$\boxed{y(t) = h + v_0 \sin\theta t - \frac{1}{2} g t^2}$$

$$v_y(t) = v_{0y} + at$$

$$\boxed{v_y(t) = v_0 \sin\theta - gt}$$

$$v_f^2 = v_i^2 + 2a\Delta y$$

split velocity



X-dir:

$$x(t) = x_0 + V_{0x}t + \tfrac{1}{2}at^2$$

$$= 0 + V_0 \cos\theta\, t$$

$$\boxed{x(t) = V_0 \cos\theta\, t}$$

$$V_{x}(t) = V_{0x} + at$$

$$\boxed{V_x(t) = V_0 \cos\theta \text{ always}}$$

a) what is the horizontal distance covered by the ball before it lands?

Plan: find the time it takes to be at $y=0$; plug this time into $x(t)$. Let's call this t_{water}

$$y(t) = h + V_0 \sin\theta\, t - \tfrac{1}{2}gt^2$$

$$0 = h + V_0 \sin\theta\, t_{water} - \tfrac{1}{2}g\, t_{water}^2$$

$\Rightarrow$ solve $at^2 + bt + c = 0$

$$\Rightarrow t_w = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a = 4.9$; $b = -28.21$; $c = -52$

$$t_w = \frac{28.21 \pm 42.6}{2 \cdot 4.9} = \boxed{7.2\,s}$$

Armed with t_w, we can find $x(t_w)$!

$x(t_w) = V_0 \cos\theta \, t_w$ = horizontal dist. traveled when it hits water.

$= (38.83)(7.2)$

$= \boxed{279.6 \text{ m}}$

b) Plan: At it's peak, the (y-component) of the velocity should be zero, so use:

$V_f^2 = V_{i,y}^2 + 2a\Delta y$

$0 = (V_0 \sin\theta)^2 - 2g\Delta y$

$\Delta y = \frac{(V_0 \sin\theta)^2}{2g} = \boxed{40.61 \text{ m}}$

So, maximum height above ocean is

$y_{max} = h + \Delta y$

$= \boxed{92.61 \text{ m}}$

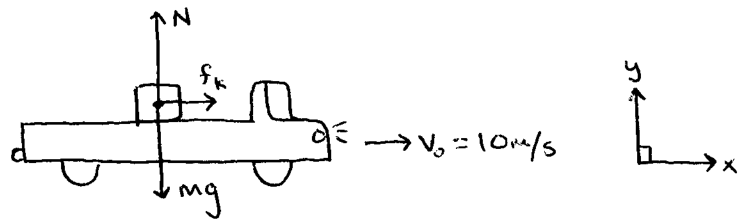
c) How long was the time of flight?

we already determined this:

$$\boxed{t_{water} = 7.2s}$$

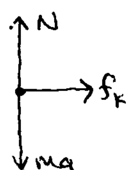
Problem 5

a) 4pts



b) 4pts

At $t=0$, the box is stationary but the truck is moving with constant velocity V_0 in the positive x-direction. To remain on the truck, the box needs to accelerate from rest to a final velocity V_0. Because the box is moving, we must use kinetic friction. Because the box is trying to "catch up" to the truck, we know that the net force must point in the positive x-direction.



Apply Newton's 2nd Law in the y-direction:

$$N - mg = 0 \rightarrow N = mg$$

From the definition of kinetic friction, we get

$$f_k = \mu_k N = \mu_k mg = 20\,N$$

The magnitude of the frictional force is $\mu_k mg$. The direction is the positive x-direction.

c) 4pts

When the box stops sliding, it is moving with constant velocity V_0. Because the box is no longer accelerating, Newton's 2nd Law tells us that there is no force in the x-direction. <u>The frictional force is zero.</u>

d) 4pts

Newton's 2nd Law: $f_k = ma \rightarrow \mu_k mg = ma \rightarrow a = \mu_k g$

Kinematics with constant acceleration: $V_f^2 - V_i^2 = 2a\Delta x$

The box is initally at rest, so $V_i = 0$

The final velocity of the box is $V_f = V_0 = 10\,m/s$. Then

$$\Delta x = V_0^2/2a = V_0^2/2\mu_k g = \frac{(10\,m/s)^2}{2 \times 0.4 \times 10\,m/s^2} = 12.5\,m$$

$$\Delta x = 12.5\,m$$

e) 4pts

The truck and box are constrained to move together, so $a_{box} = a_{truck} = -5\,m/s^2$. Static friction causes the box to decelerate, so $f_s = ma = -(5\,kg)(5\,m/s^2) = -25\,N$. So the static friction has magnitude 25 N and points to the left.

Check: $|f_s| \leq f_{s,max} = \mu_s N = \mu_s mg = 30\,N$