## Problem \#1 Discrete Fourier Transform (DFT) (40 points)

As discussed in class and in the DFT handout, the block diagram below represents the operations involved in finding the DFT of a time signal $x(t)$.

Recall that the DFT of length $N$ sequence $\mathrm{x}[n]$ is $\mathrm{X}[k]=\operatorname{sum}\left(\mathrm{x}[\mathrm{n}] * e^{\wedge}(-j * 2 * p i * n * k / N), n=0, n=N-1\right) . \mathrm{X}[k]$ values can be found by using the block diagram
to find $\mathrm{X}^{\prime}(\mathrm{w})$ and then noting that $\mathrm{X}[k]=\mathrm{To} /(2 * \mathrm{pi})^{*}$ area $\left\{\mathrm{X}^{\prime}\left(k^{*} 2 * \mathrm{p} / / \mathrm{To}\right)\right\}$. For each part below, with $\mathrm{Ts}=1$ second, $\mathrm{To}=16$ seconds, match the time samples
$\mathrm{x}[n]$ ( $=\operatorname{area}\left\{\mathrm{x}^{\prime}(\mathrm{nTs})\right\}$ (1 pt.each) and DFT samples $\mathrm{X}[k]$ (4 pts. each) with the time signal $x(t)$.
The window function: $\mathrm{w}(\mathrm{t})=\{0$ if $t<0-, 1$ if $0-<=t<16-, 0$ if $16-<=t$
Hint: $w(t) \sim=\operatorname{PI}[(t-T o / 2) / T o]$
For each part, pick the letter of the matching plot from the next pages. Hint \#1: All X[k] are real. Hint \#2: $\mathrm{x}[n]$ are A-H and X[k] are IP.

a) $\mathrm{x} 1(t)=\cos \left(\mathrm{pi}^{*} t / 8\right)$
---->x1[n] is plot:
---->X1[k] is plot:
b) $\mathrm{x} 2\left(t 0=\cos \left(\mathrm{pi}^{*} t / 4\right)\right.$
---->x2[n] is plot:
---->X2[k] is plot:
c) $\mathrm{x} 3(t)=\cos \left(\mathrm{pi}^{*} t / 2\right)$
---->x3[n] is plot:
---->X3[k] is plot:
d) $\mathrm{x} 4[n]=\left\{\cos \left(n^{*} \mathrm{pi} / 8\right)\right.$ if $n$ even, 0 if $n$ odd
---->x4[n] is plot:
---->X4[k] is plot:

e) $x 5(t)=$
---->x5[n] is plot:
---->X5[k] is plot:
f) $\mathrm{x} 6(t)=\cos \left[\mathrm{pi} / 3^{*}(t-\mathrm{To} / 2)\right]$
---->x6[ $n$ ] is plot:
---->X6[k] is plot:

g) $x 7(t)=$
---->x7[n] is plot:
---->X7[k] is plot:

h) $x 8(t)=$
---->x8[ $n$ ] is plot:
---->X8[k] is plot:

## Plots for Prob 1:

A


C


E


G


I


K


M


0






J


L




## Problem \#2 (42 points)

Let $m(t)=\frac{\sin 1000 \pi t}{1000 \pi t} . \quad F\{m(t)\}=M(\omega)=\Pi\left(\frac{\omega}{2000 \pi}\right)$

(10 pts.) a) Let $\mathrm{x} 1(t)=m(t) \cos \left(4000^{*} \mathrm{pi}^{*} t\right)$. Sketch $\mathrm{x} 1(t)$ for $0<=t<=2 * 10^{\wedge}-3 \mathrm{sec}$, labeling peak height and accurately indicating zero crossings.

Sketch $\mathrm{X} 1(\mathrm{w})$, labeling height/area, center frequency, and sideband frequencies.
(10 pts.) b) Let $\mathrm{x} 2(t)=(1+m(t)) \cos \left(4000^{*} \mathrm{pi}{ }^{*} \mathrm{t}\right)$. Sketch $\mathrm{x} 2(t)$ for $0<=\mathrm{t}<=2^{*} 10^{\wedge}-3$ sec, labeling peak height and accurately indicating zero crossings.

Sketch X2(w), labeling height/area, center frequency, and sideband frequencies.
$(6 \mathrm{pts}.) \mathbf{c}) \times 2(t)=(1+m(t)) \cos (4000 * \mathrm{pi} * \mathrm{t})$ is passed through the system shown, with $\mathrm{wf}=1500 * \mathrm{pi}$


Hint: $\mathrm{S}(\mathrm{w})=\operatorname{sum}\left(\left(2 * \sin \left(k^{*} \mathrm{pi} / 2\right) / k\right) * \operatorname{delta}\left(\mathrm{w}-4000^{*} \mathrm{pi} * k\right)\right), k=-$ infinity to + infinity $)$
Sketch $Z(w)$, labeling height area/area, center frequencies, and sideband frequencies for $0<=w<=12000 * \mathrm{pi}$
[Hint: $\mathrm{Z}(\mathrm{w})$ is real and even]
$(\mathbf{8} \mathbf{~ p t s .}) \mathbf{d}) \times 1(t)=m(t) \cos \left(4000^{*} \mathrm{pi}^{*} t\right)$ is passed through an ideal diode and ideal low pass filter $H(\mathrm{w})$, with $\mathrm{wf}=1500^{*} \mathrm{pi}$


Approximately sketch $v(t)$, for $0<=t<=2 * 10^{\wedge}-3$ sec

Is $v(t)$ an accurate estimate of the form of $m(t)$ ? Why or why not?
(8 pts.) e) Let $\mathrm{x} 3(t)=m(t) \cos \left(4000^{*} \mathrm{pi}^{*} t\right)+(m(t) * 1 /(\mathrm{pi} * \mathrm{t})) \sin \left(4000^{*} \mathrm{pi}^{*} t\right)$. Sketch $\mathrm{X} 3(\mathrm{w})$, labeling height/area, center frequency, and sideband frequencies.

## Problem \#3 (12 points)

You are given modulation $m(t)=\cos \left(1000^{*} \mathrm{pi}^{*} t\right)$, which drives 4 transmitters $\mathrm{a}, \mathrm{b}$, c , and d. Each transmietter generates
new signals $a(t), b(t), c(t), d(t)$ with spectra $\mathrm{A}(\mathrm{w}), \mathrm{B}(\mathrm{w}), \mathrm{C}(\mathrm{w}), \mathrm{D}(\mathrm{w})$ using $m(t)$, as shown below:


For each transmitter, identify the modulation method used (for example, AM - DSB - LC, WB FM, etc.), the
channel bandwidth used, and the power efficiency (fraction of power transmitting useful information).

|  | Modulation Method | Channel BW | Power Efficiency |
| :--- | :--- | :--- | :--- |
| $\mathbf{A}(\mathbf{w})$ |  |  |  |
| $\mathbf{B}(\mathbf{w})$ |  |  |  |
| $\mathbf{C}(\mathbf{w})$ |  |  |  |
| $\mathbf{D}(\mathbf{w})$ |  |  |  |

## Problem \#4 (6 points)

The Federal Communications Commission has assigned you a transmission channel from 1.000 MHz to
1.010 MHz. You have two bandlimited message signals $m 1(t)$ and $m 2(t)$ with spectra $M 1(w)$ and $M 2(w)$, as shown:


Draw a block diagram for a transmitter that could send both messages simultaneously in the given channel bandwidth.

Your transmitter contains multiplier blocks, oscillators, and summers only.

## Posted by HKN (Electrical Engineering and Computer Science Honor Society) <br> University of California at Berkeley <br> If you have any questions about these online exams <br> please contact mailto:examfile@hkn.eecs.berkeley.edu

