# EECS 120, Fall 1993 <br> Final <br> Professor Fearing 

## Problem \#1 (23 points)

Classify the following systems. In each column, write "yes", "no", or "?" (use "?" if not decidable with given information). The input to the system is $\mathrm{x}(\mathrm{t})$ and the output is $\mathrm{y}(\mathrm{t})$. (Note: do not fill in an answer in the blacked out space for letter (e)/BIBO stable.)

|  | Causal | Linear | Time-invariant | BIBO stable |
| :--- | :--- | :--- | :--- | :--- | :--- |
| a. $\mathrm{y}(\mathrm{t})=2 \mathrm{x}(\mathrm{t})+1$ |  |  |  |  |
| b. $\mathrm{y}^{\wedge}(\mathrm{t})+\mathrm{y}(\mathrm{t})=\mathrm{tx}(\mathrm{t})$ |  |  |  |  |
| c. $\mathrm{y}(\mathrm{t})=\mathrm{x}(\mathrm{t}) \cos \left(\mathrm{w} \_\mathrm{c}^{*} \mathrm{t}\right)$ |  |  |  |  |
| d. $\mathrm{Y}(\mathrm{w})=\mathrm{X}^{\wedge} 2(\mathrm{w}), \mathrm{x}(\mathrm{t})=0$ for $(\mathrm{t}<0)$ |  |  |  |  |
| e. $\mathrm{y}^{\wedge} 2(\mathrm{t})+\mathrm{y}(\mathrm{t})=\mathrm{x}(\mathrm{t}+2)$ |  |  |  |  |
| f. $\mathrm{y}(\mathrm{t})=\mathrm{h}(\mathrm{t}) * \mathrm{x}(\mathrm{t}) * \mathrm{~d}(\mathrm{t}+2)$ |  |  |  |  |

## Problem \#2 (12 points)

Which of the following are eigen functions for LTI systems? (Recall: $\mathrm{x}(\mathrm{t})$ is an eigen function if $\mathrm{h}(\mathrm{t}) * \mathrm{x}$ $(\mathrm{t})=\mathrm{A} * \mathrm{x}(\mathrm{t})$, where A is a coplex constant.) Circle the boldface letter(s) for which the above is true.
a. $\sin \left(w \_0 * t\right) u(t)$
b. $\mathrm{e}^{\wedge}\left(\mathrm{s} \_0 * \mathrm{t}\right)$
c. $t^{\wedge}(1 / 2)$
d. $t+1$
e. $\cos \left(\mathrm{w} \_0 * \mathrm{t}\right)+\sin \left(\mathrm{w} \_0 * \mathrm{t}\right)$
f. $\sin \left(\mathrm{w} \_1 * \mathrm{t}\right)+\sin \left(\mathrm{w} \_2 * \mathrm{t}\right)$

## Problem \#3 (10 points)

[5 pts.] a) Consider the following block diagram:


Determine the transfer function for the system:
$\mathrm{H}(\mathrm{s})=\mathrm{Y}(\mathrm{s}) / \mathrm{X}(\mathrm{s})=$ $\qquad$
[5 pts.] b) A system has transfer function $\mathrm{H}(\mathrm{s})=4+\left[(2 \mathrm{~s}+3) /\left(\mathrm{s}^{\wedge} 2+6 \mathrm{~s}+9\right)\right]$.
What is the impulse response? $\mathrm{h}(\mathrm{t})=$ $\qquad$ Is this system BIBO stable?

## Problem \#4 (20 points)

A signal $x(t)$ is corrupted by an echo. The observed signal $y(t)$ then has the form: $\mathrm{y}(\mathrm{t})=\mathrm{x}(\mathrm{t})+\mathrm{kx}\left(\mathrm{t}-\mathrm{T} \_0\right) ;|\mathrm{k}|<1$
[10 pts.] a) Find $H(s)$, the transfer function for a linear system which will yield $x(t)$ as its output if $y(t)$ is the input ("echo cancelling").

$\mathrm{H}(\mathrm{s})=$ $\qquad$
[8 pts.] b) Sketch

[2 pts.] c) [Harder] Consider the pole locations for $\mathrm{H}(\mathrm{s})$. Is $\mathrm{H}(\mathrm{s})$ BIBO stable? If yes, explain why. If no, find a bounded input $y(t)$ which gives rise to an unbounded output $x(t)$.

## Problem \#5 (30 points)

You receive an AM (DSB-LC) signal with spectrum X(w):
$X(\omega)$;

[15 pts.] a) Sketch $x(t)$ in the range $0<=\mathrm{t}<=5^{*} 10^{\wedge}-3 \mathrm{sec}$. (Specify amplitude at 0 sec . and 5 msec .)

[15 pts.] b) The message is received using asynchronous detection in the following system. Sketch Y (w), the spectrum after detection by the ideal diode, in the range -3000 (pi) $<=\mathrm{w}<=3000$ (pi)

Hint: $2(t)$ has spectrum:


Sketch $\mathrm{Y}(\mathrm{w})$, indicating important amplitudes and frequencies.


## Problem \#6 (15 points)

The impulse response $\mathrm{h}[\mathrm{n}]$ of a discrete time LTI system is given:


Find $g[n]$, the impulse response of the inverse system, i.e., find a $g[n]$ such that $g[n] * h[n]=d[n]$. (Note: $\mathrm{g}[\mathrm{n}]$ should be in closed form.)
$\mathrm{d}[\mathrm{n}]$---> [h[n]] ---> [g[n] ] ---> d[n]
$\mathrm{g}[\mathrm{n}]=$ $\qquad$

## Problem \#7 (20 points)

A system is described by the following differential equation with input $x(t)$ and output $y(t)$ : $\left(d^{\wedge} 2 \mathrm{y} / \mathrm{dt} \wedge 2\right)+\mathrm{y}=(\mathrm{d} / \mathrm{dt}) \mathrm{x}$
[5 pts.] a) Conver this differential equation to a defference equation using the backward difference approximation to the derivative, i.e., $\mathrm{dx} / \mathrm{dt}=\sim(\mathrm{x}[\mathrm{n}]-\mathrm{x}[\mathrm{n}-1]) / \mathrm{T}$. Assume sample rate $\mathrm{T}=0.5 \mathrm{sec}$.
[5 pts.] b) Given the following difference equation, find $\mathrm{H}(\mathrm{z})$, the z -transform of the unit pulse response
$[\mathrm{n}-2]+3 \mathrm{y}[\mathrm{n}-1]+2 \mathrm{y}[\mathrm{n}]=\mathrm{x}[\mathrm{n}-1]$
$\mathrm{H}(\mathrm{z})=$ $\qquad$
[5 pts.] c) For the difference equation in part b , above, find $\mathrm{y}[\mathrm{n}]$ for $\mathrm{x}[\mathrm{n}]=0$,
$(\mathrm{ZIR})$ with $\mathrm{y}[-1]=1, \mathrm{y}[-2]=0$.
(Note: $\mathrm{y}[\mathrm{n}]$ should be in closed-form.)
$\mathrm{y}[\mathrm{n}]=$ $\qquad$
[5 pts.] d) For the difference equation in part $b$, find $y[n]$ for $x[n]=u[n]$, (ZSR) with $\mathrm{y}[-1]=0$ and $\mathrm{y}[-2]=0$.
$\mathrm{y}[\mathrm{n}]=$ $\qquad$

## Problem \#8 (15 points)

A continuous time system has impulse response $h(t)=u(t)$.
[5 pts.] a) Determine the equivalent discrete time filter $\mathrm{H}(\mathrm{z})$ using the Impulse Invariance method using sampling rate $\mathrm{T}=1.0$ second.
H_1(z) = $\qquad$
[5 pts.] b) Sketch the magnitude of the frequency response for the continuous time and discrete time filters:

$\left|H_{1}\left(e^{j \omega T}\right)\right|$

[5 pts.] Explain the reason(s) for differences (if any) between the two sketches in part b.

## Problem \#9 (24 points)

For each pole-zero diagram belo, fill in the blank with the letter corresponding to the closest phase spectrum below. (Spectrums may be used more than once.) (Note: The angle function is defined from (pi) to $+(\mathrm{pi})$.) Assume $\mathrm{T}=1.0$.


Prese Fesponses:
A



D



$\sigma$



## Problem \#10 (16 points)

[8 pts.] a) Sketch a pole-zero diagram in the z-plane for a stable, causal FIR filter with the following magnitude spectrum. You may make reasonable engineering approximations for your diagram. (Hint: The filter has four poles.) Don't worry about the exact height at 0 or (pi). You may leave locations in terms of parameter p .

[8 pts.] b) Determine the unit pulse response h[n] of the FIR filter. (Hint: The filter has 4 poles.)
$\mathrm{h}[\mathrm{n}]=$ $\qquad$

## Posted by HKN (Electrical Engineering and Computer Science Honor Society) University of California at Berkeley <br> If you have any questions about these online exams please contact mailto:examfile@hkn.eecs.berkeley.edu

