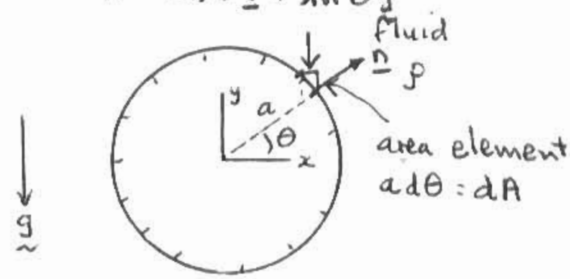
MEAN	137/200
SD	40/200

ME106 Fluid mechanics
1st Test, S04

NAME SOLUTIONS

$\hat{n} = \cos\theta\, \underline{i} + \sin\theta\, \underline{j}$

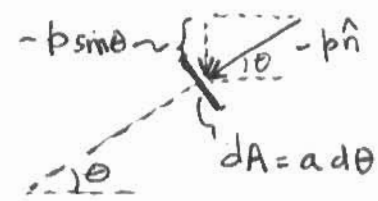


MEAN 43
SD 21

1. (60) (a) The cylinder of radius a is at rest in motionless fluid of density ρ. By integrating the hydrostatic pressure force over the cylinder surface, find the buoyant force exerted by the fluid on the cylinder. Verify that your answer is consistent with Archimedes's principle. GIVEN $\int_0^{2\pi} \sin^2\theta \, d\theta = \pi$

Forces are per unit length of cylinder

(30) Vertical force acting on the element $a\,d\theta$ is $\underbrace{-\,p\,\underbrace{(a\,d\theta)}_{dA}\,\underbrace{(\sin\theta)}_{n_y}}$ (+10)



Resultant vertical force $\quad F = -a\int_0^{2\pi} p\sin\theta\, d\theta$

$$= a\int_0^{2\pi} (p - p_0)\sin\theta\, d\theta \quad \because \text{const. pressure } p_0 \text{ gives no resultant force on body}$$

p_0, pressure at $y = 0$

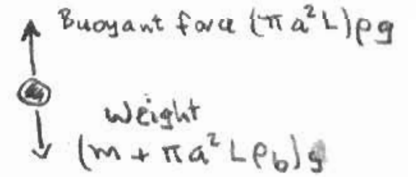
But $\quad \boxed{p - p_0 = -\rho g y = -\rho g a \sin\theta}$ +10

$$\therefore \quad \boxed{F = \rho g a^2 \int_0^{2\pi} \sin^2\theta\, d\theta = \pi \rho g a^2}$$

RESULT (+)ve (upwards force) +10

Compatible with Archimedes's princ. $\quad F = (\pi a^2)\rho g$ = weight displaced p.u. length of cylinder

(30) **(b)** A blimp in the form of a cylinder of length L and radius a is to lift a mass m. Find a as a function of m, g, L, ρ and the density ρ_b of the gas in the blimp. Hence find a for $m = 100$ kg, $L = 20$ m, $\rho = 1.2$ kg/m^3 and $\rho_b = 0.17$ kg/m^3 (Helium).



$$mg + \pi a^2 L g \rho_b = \pi a^2 L g \rho$$

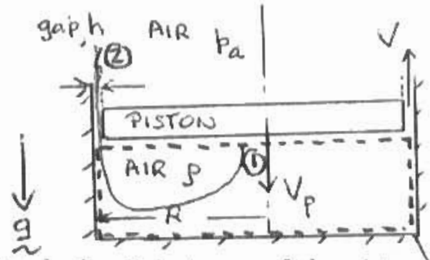
$$\Rightarrow a = \left(\frac{m}{\pi(\rho - \rho_b) L} \right)^{1/2}$$

$$= \left(\frac{100}{\pi(1.2 - 0.17)\, 20} \right)^{1/2} = \left(\frac{5}{\pi\,(1.2 - 0.17)} \right)^{1/2} = \underline{1.2 \text{ m}}$$

DISTRIBUTION	
180-200	14
160-179	9
140-159	17
120-139	10
100-119	6
80-99	6
60-79	4
40-59	2
	68

PLEASE PRINT YOUR NAME ON THIS PAGE

MEAN 54
SD 18



2. **(70)** A piston of mass M sinks under its own weight at constant velocity V_p into an air–filled cylinder. The air displaced by the moving piston escapes through the narrow gap between the piston and wall to form a free jet, as shown in the sketch.

(a) By balancing mass on the control volume shown, express the jet velocity V in terms of the piston velocity V_p, gap width h and piston radius R. GIVEN $R \gg h$

control volume

$$\underbrace{V_p \cancel{\pi} (R-h)^2}_{\text{inflow below piston}} = V \cancel{\pi} \left(R^2 - (R-h)^2 \right) \quad] + 20$$

$$(R + (R+h))(R - (R-h)) \simeq 2Rh, \; h \ll R$$

30

$$\therefore \quad \boxed{V \simeq \frac{R}{2h} V_p} \qquad R \gg h$$

−10 MINOR SLIPS ON AREA.

(b) Assuming the flow to be steady and inviscid, and taking the effect of gravity on the pressure field as negligible, find the pressure p acting on the base of the piston.

B.E. along SL $1 \to 2$ SHOW S/LS ON SKETCH! (ABOVE) (Must end on piston, as in draining tank, LECT 8.)

25

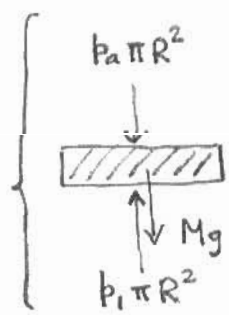
$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2$$

$$V_1 \ll V_2 \Rightarrow \quad p_1 - p_a = \tfrac{1}{2}\rho V_p^2 \frac{R^2}{4h^2} \Rightarrow \quad \boxed{p_1 - p_a = \frac{1}{8}\rho V_p^2 \frac{R^2}{h^2}}$$

$p_2 = p_a$, FREE JET
$V_2 = V$ +10

RESULT +15

(c) Hence find the piston velocity V_p in terms of M, g, h, R, and the air density ρ.



Force balance on piston $\Sigma F_z = 0$ +5

$$\Rightarrow \; Mg = (p_1 - p_a)\pi R^2 = \frac{1}{8}\pi \rho V_p^2 \frac{R^4}{h^2} \quad +5$$

15

$$\Rightarrow \; \boxed{V_p = \left(\frac{8}{\pi} \frac{Mgh^2}{\rho R^4} \right)^{1/2}} \qquad \text{RESULT } +5$$

CHECK THAT YOUR RESULTS ARE DIMENSIONALLY CORRECT

$$LT^{-1} \qquad \left(\frac{\cancel{M} LT^{-2} L^2}{\cancel{M} L^{-3} L^4} \right)^{1/2} = LT^{-1} \; ✓$$

CORRECT APPROACH WITH MINOR SLIPS
−5 ALL SLIPS

MEAN 40
SD 14

$[k] = L^2 T^{-1}$

3. (70) The velocity field $\mathbf{V} = k(x\mathbf{i} + y\mathbf{j})/r^2$, ($k > 0$ constant, and $r^2 = x^2 + y^2$), represents an inviscid flow.

(a) Find the streamlines, and sketch them.

40

$$\underbrace{\frac{dx}{u} = \frac{dy}{v}}, \quad u = \frac{kx}{r^2}, \quad v = \frac{ky}{r^2} \qquad +25$$

CHECK

$y = \frac{y_0}{x_0} x$

$\Rightarrow \frac{dy}{dx} = \frac{y_0}{x_0} = \frac{y}{x}$ ✓

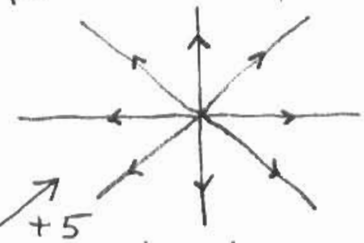
DE is satisfied

$$\Rightarrow \underbrace{\frac{dx}{x} = \frac{dy}{y}}$$

$$\Rightarrow d \ln\left(\frac{x}{x_0}\right) = d \ln\left(\frac{y}{y_0}\right)$$

$$\Rightarrow \boxed{\frac{x}{x_0} = \frac{y}{y_0}}$$

Rays through origin +10



+5

streamlines

(b) Use the streamwise and normal components of Euler's equations of motion to find $\partial p/\partial s$ and $\partial p/\partial n$. Integrate these equations to find the pressure p.

25

normal $\quad \rho \frac{V^2}{R} = -\frac{\partial p}{\partial n} \qquad$ n inc. to centre of curvature

Here $R = \infty$, streamlines have zero curvature.

$$\Rightarrow \boxed{\frac{\partial p}{\partial n} = 0} \quad +10$$

streamwise $\quad \rho V \frac{\partial V}{\partial s} = -\frac{\partial p}{\partial s}$, $\quad s = r$ increases in streamwise direct.

$$V = |\mathbf{V}| = \left[k^2 \left(\frac{x^2+y^2}{r^4}\right)\right]^{1/2} = \frac{k}{r}$$

$$+10 \quad \boxed{\frac{\partial p}{\partial r} = -\rho \frac{k}{r}\left(-\frac{k}{r^2}\right) = \rho k^2 r^{-3}}$$

RESULT +5 $\Rightarrow \boxed{p = -\frac{1}{2}\rho k^2 r^{-2} + \text{const}}$ $\quad$ (No fn of n $\because \frac{\partial p}{\partial n} = 0$.)

DIMENSIONS: $[p] = M L^{-1} T^{-2} \qquad [\rho k^2 r^{-2}] = M L^{-3} \; L^4 T^{-2} \; L^{-2} = M L^{-1} T^{-2}$ ✓

5

(c) Give an example of a steady incompressible inviscid flow in which $\frac{1}{2}\rho V^2 + p + \rho g z$ is **not** constant across streamlines.

Rigid rotation with $V = \Omega r$

Circular streamlines; both p and V increase with r so $p + \frac{1}{2}\rho V^2$ not const across SLs.

SEE READER LECT 8, 9

CHECK THAT YOUR RESULTS ARE DIMENSIONALLY CORRECT

END

Example must be inviscid (no sudden expansion!)

Jet with no mixing into still fluid (LECT 8) also acceptable.