EECS 40, Spring 2007
Prof. Chang-Hasnain
Test #2

October 8, 2007
Total Time Allotted: 50 minutes
Total Points: 100

1. This is a closed book exam. However, you are allowed to bring one page (8.5" x 11"), single-sided notes.
2. No electronic devices, i.e. calculators, cell phones, computers, etc.
3. SHOW all the steps on the exam. Answers without steps will be given only a small percentage of credits. Partial credits will be given if you have proper steps but no final answers.
4. Draw BOXES around your final answers.
5. **Remember to put down units. 1 point will be taken off per missed unit.**

Last (Family) Name:____

First Name: ____

Student ID: ______________________ **LAB Session**: ____________

Signature: ____

Score:	
Problem 1 (16 pts)	
Problem 2 (28 pts):	
Problem 3 (56 pts):	
Total	

1. (16 pts) Phasors and complex numbers

a) (6 pts) Convert $\vec{V} =$ V to phasor notation. **Both polar and exponential form are acceptable.**

$$\vec{V} = \frac{4\sqrt{2} - j8}{2 - j2\sqrt{2}} = \frac{4\sqrt{2}(1 - j\sqrt{2})}{2(1 - j\sqrt{2})} = 2\sqrt{2}\angle 0\text{ V} \;\; \text{(pink)}$$

$$\vec{V} = \frac{4\sqrt{2} + j8}{2 + j2\sqrt{2}} = \frac{4\sqrt{2}(1 + j\sqrt{2})}{2(1 + j\sqrt{2})} = 2\sqrt{2}\angle 0\text{ V} \;\; \text{(white)}$$

$$\vec{V} = \frac{4\sqrt{2} - j8}{2 - j2\sqrt{2}} = \frac{4\sqrt{2}(1 - \sqrt{2})}{2(1 - j\sqrt{2})} = 2\sqrt{2}\angle 0 \;\; \text{V (yellow)}$$

b) (3 pts) What is v(t), in **cosinusoidal** form? Assume frequency is ω.

$$v(t) = 2\sqrt{2}\cos(\omega t)\text{ V (all)}$$

c) (4 pts) Convert $\vec{V_2}$ V to phasor notation. **Both polar and exponential form are acceptable.**

$$\vec{V}_2 = j = e^{j\frac{\pi}{2}} = 1\angle 90\text{ V (yellow and pink)}$$

$$\vec{V}_2 = -j = e^{-j\frac{\pi}{2}} = 1\angle -90\text{ V (white)}$$

d) (3 pts) What is $v_2(t)$, in **cosinusoidal** form? Assume frequency is ω.

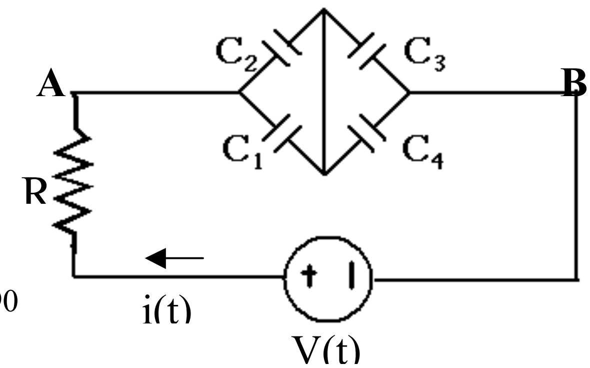
$$v_2(t) = \cos(\omega t + \frac{\pi}{2})\text{ V (yellow and pink)}$$

$$v_2(t) = \cos(\omega t - \frac{\pi}{2})\text{ V (white)}$$

2. (28 pts) Complex impedance.

a) (8 pts) C_1=C_2=C_3=C_4=μF, R= Ohm $V(t) = 10\cos(2\pi ft)$ V, frequency $f = \frac{10^6}{2\pi}$ Hz

What is the equivalent impedance $\overrightarrow{Z_{AB}}$?



$C_{eq} = (C_1 + C_2) \| (C_3 + C_4) = (2\mu F) \| (2\mu F) = 1\mu F$
(yellow)

$\overrightarrow{Z_{AB}} = \frac{1}{j\omega C} = \frac{1}{j(10^6 rads/s)(10^{-6} F)} = \frac{1}{j}\Omega = 1\angle -90$
(yellow)

$C_{eq} = (C_1 + C_2) \| (C_3 + C_4) = (4\mu F) \| (4\mu F) = 2\mu F$ (white and pink)

$\overrightarrow{Z_{AB}} = \frac{1}{j\omega C} = \frac{1}{j(10^6 rads/s)(2\times 10^{-6} F)} = \frac{1}{2j}\Omega = -\frac{j}{2}\Omega$(white and pink)

b) (6 pts) What is the current i(t) in phasor form?

$\overrightarrow{Z_{eq}} = R + C_{eq} = 1 + \frac{1}{j}\Omega = 1 - j\Omega = \sqrt{2}\angle -45^\circ$ (yellow)

$\overrightarrow{Z_{eq}} = R + C_{eq} = 2 + \frac{1}{2j}\Omega = 2 - \frac{j}{2}\Omega = \frac{\sqrt{17}}{2}\angle \tan^{-1}(-\frac{1}{4})$ (white and pink)

$\vec{V} = 10\angle 0^\circ$

$\vec{I} = \frac{\vec{V}}{\overrightarrow{Z_{eq}}} = \frac{10\angle 0}{\sqrt{2}\angle -45} = 5\sqrt{2}\angle 45^\circ\, A$ (yellow)

$\vec{I} = \frac{\vec{V}}{\overrightarrow{Z_{eq}}} = \frac{10\angle 0}{\frac{\sqrt{17}}{2}\angle \tan^{-1}(-\frac{1}{4})} = \frac{20}{\sqrt{17}}\angle -\tan^{-1}(-\frac{1}{4})\, A$ (white and pink)

c) (8 pts) Now replace the four capacitors with inductors L_1= L_2= L_3= L_4=1 μH and calculate the equivalent impedance $\overrightarrow{Z_{AB}}$.

$L_{eq} = (L_1 \| L_2) + (L_3 + L_4) = (.5) + (.5) = 1\ \mu$H (yellow)

$L_{eq} = (L_1 \| L_2) + (L_3 + L_4) = (1) + (1) = 2\mu H$ (white and pink)

$\overrightarrow{Z_{AB}} = j\omega L_{eq} = j(10^6 rads/s)(10^{-6} H) = j\Omega$ (yellow)

$\overrightarrow{Z_{AB}} = j\omega L_{eq} = j(10^6 rads/s)(2\times 10^{-6} H) = 2j\Omega$ (white and pink)

d) (6 pts) In this case, what is the current i(t) in phasor form?

$$\vec{V} = 10\angle 0^\circ \text{ (all)}$$

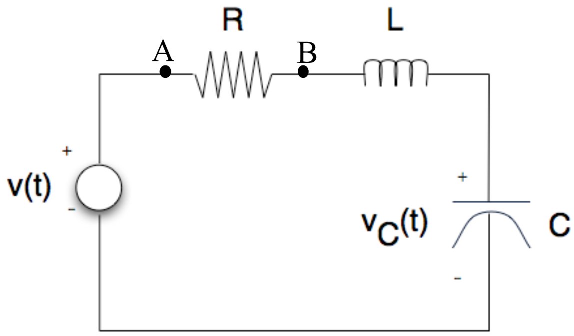
$$\overrightarrow{Z_{eq}} = R + L_{eq} = 1 + j\Omega = \sqrt{2}\angle 45^\circ \text{ (yellow)}$$

$$\vec{I} = \frac{\vec{V}}{\overrightarrow{Z_{eq}}} = \frac{10\angle 0}{\sqrt{2}\angle 45} = 5\sqrt{2}\angle -45^\circ \text{ (yellow)}$$

$$\overrightarrow{Z_{eq}} = R + L_{eq} = 2 + 2j\Omega = 2\sqrt{2}\angle 45^\circ \text{ (white and pink)}$$

$$\vec{I} = \frac{\vec{V}}{\overrightarrow{Z_{eq}}} = \frac{10\angle 0}{2\sqrt{2}\angle 45} = \frac{5\sqrt{2}}{2}\angle -45^\circ \text{ (white and pink)}$$

3. (56 pts) We have a circuit with R, L, C and v(t) as an input.



(a) (16 pts) If $v_C(t)$ is the voltage across the capacitor C, we can formulate the 2nd order circuit as follows.

$$\frac{d^2 v_c(t)}{dt^2} + A\frac{dv_c(t)}{dt} + Bv_c(t) = f(t)$$

What are A,B, and f(t)? Express them in terms of R,L,C and v(t).

$$A = \frac{R}{L}$$

$$B = \frac{1}{LC}$$

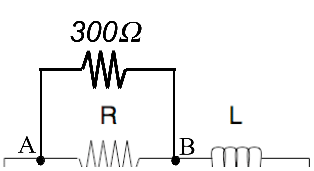
$$f(t) = \frac{v(t)}{LC}$$

(b) (5 pts) The undamped resonance frequency $\square_0$ is $\square_{0}$=10^4 Hz and L is 10mH, what is the value of C?

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{\omega_o{}^2 L} = \frac{1}{(10^4 rad/s)^2(.01H)} = 10^{-6}F$$

(c) (10 pts) By adding another 300 Ohm resistor in parallel to the R, connecting at points A and B, see Figure below, we find the circuit is critically damped. What is the value of R? (NOTE: If you did not get the value for C from part b, full credit awarded for solution including C as a variable.)



$\zeta = 1$

Thus:

$\alpha = \omega_o$

$\frac{R_{eq}}{2L} = 10^4 rad / s$

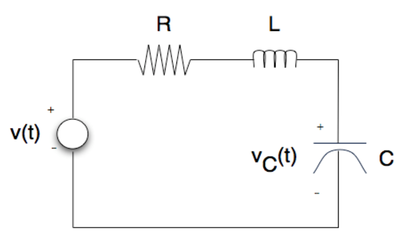
$R_{eq} = 2L(10^4 rad / s) = 2(.01H)(10^4 rad / s) = 200\Omega$

$R_{eq} = \frac{R(300)}{R+300} = 200\Omega$

$R = \frac{2}{3}(R+300)$

$R = 600\Omega$

(d) (5 pts) Is the original circuit (without the 300 Ohm parallel resistor, see Figure below) overdamped or underdamped?



$\zeta = \frac{\alpha}{\omega_o}$

Thus:

$\alpha = \frac{R}{2L} = \frac{600}{2(.01H)} = 30000$

$\omega_o = 10^4 rad/s$

$\zeta = 3$

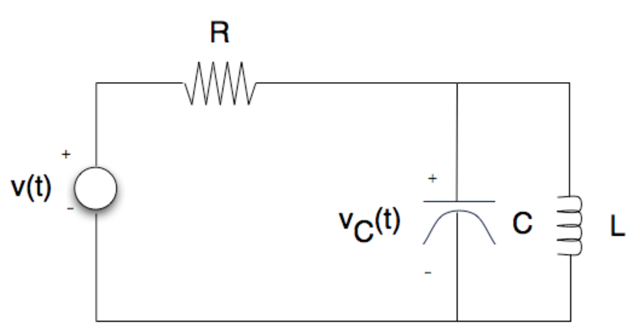
Thus the original circuit is overdamped.

Intuitively, the original circuit has a higher resistance, thus more energy is lost across the resistor, damping the circuit more than the critically damped case.

(e) (20 pts) We change the configuration of L and C to be in parallel as shown below, with the original values for R, L and C – the values you got from parts a- c.

What is the resonance frequency ω_0?
What is the damping ratio ζ ?
Is this circuit under-, critically, or over- damped?

(NOTE: If you did not get the values of RLC from parts a-c, you will get full credit if you can give all possible if-then's.)



If a Thevenin to Norton conversion is performed on the voltage source and resistor, it becomes apparent that this is a parallel RLC circuit. We then use the equations for a parallel RLC circuit:

$\omega_o = \frac{1}{\sqrt{LC}} = 10^4 rad/s$ (same as in the series case)

$$\alpha = \frac{1}{2RC} = \frac{1}{2(600\Omega)(1\mu F)} = \frac{2500}{3}$$

$$\zeta = \frac{\alpha}{\omega_o} = \frac{2500}{3(10^4)} = \frac{1}{12}$$

$\zeta < 1$, so the circuit is underdamped.