1. Short-Channel MOSFET Model [17 points].

Device parameters:

\[ C_{ox} = 4 \, \text{fF/\mu m}^2 \]
\[ W = 2.5 \, \text{\mu m} \]
\[ L = 0.1 \, \text{\mu m} \]
\[ V_{Th} = 1 \, \text{V} \]
\[ V_{DS,sat} = 0.75 \, \text{V} \]
\[ \lambda_n = 0.05 \, \text{V}^{-1} \]
\[ v_{sat} = 10^2 \, \text{cm/s} \]

An improved model for the velocity-saturated MOSFET is:

\[
i_D = C_{ox} W v_{sat} (v_{GS} - V_{Th})(v_{DS} - \frac{V_{DS}}{V_{DS,sat}})
\times \left(1 - \frac{v_{DS}}{2V_{DS,sat}}\right)\quad \text{when } v_{DS} \leq V_{DS,sat} = 0.75 \, \text{V (triode region)}
\]

\[
i_D = \left(\frac{1}{2}\right) C_{ox} W v_{sat} (v_{GS} - V_{Th}) \left[1 + \lambda_n v_{DS} \right] \left[\frac{1 + \lambda_n v_{DS}}{1 + \lambda_n V_{DS,sat}}\right]
\quad \text{when } v_{DS} > V_{DS,sat} = 0.75 \, \text{V (saturation region)}
\]

The drain characteristics for this short-channel MOSFET model are:

![Graph showing drain current (i_D) vs. gate-source voltage (v_{GS}) for different source-drain voltages (v_{DS}).]
(a) [4 pts.] What is the small-signal transconductance $g_m$ at the operating point $Q_1$ in $\mu S$? You can find the answer either from the drain current equations or graphically: in either case, be sure to explain your method clearly.

(b) [4 pts] What is the small-signal drain resistance $r_d$ at the operating point $Q_1$ in $k\Omega$? For this parameter at this operating point, graphical techniques don’t give a sufficiently accurate answer.
(c) [4 pts.] What is the transconductance $g_m$ at the operating point $Q_2$ in $\mu S$? You can find the answer either from the drain current equations or graphically: in either case, be sure to explain your method clearly.

(d) [4 pts] What is the small-signal drain resistance $r_o$ at the operating point $Q_2$ in $k\Omega$? You can find the answer either from the drain current equations or graphically: in either case, be sure to explain your method clearly.
2. BJT voltage buffer [18 pts.]

Given:
\[ \beta_o = 100 \]
\[ V_{th} = 25 \text{ mV} \]
\[ V_A = 50 \text{ V} \]
\[ V_{CEsat} = 0.1 \text{ V} \]
\[ R_S = 3 \text{ k}\Omega \]
\[ R_E = 5 \text{ k}\Omega \]
\[ R_L = 2 \text{ k}\Omega \]

(a) [3 pts.] Find the numerical value of \( V_B \) such that \( V_{OUT} = 2.5 \text{ V} \). Your answer should be accurate to +/- 5%. Notes: (i) the gray boxes indicate small-signal elements that can be neglected for the DC bias analysis and (ii) the DC base current \( I_B \) of the bipolar transistor can be neglected for the bias solution.

(b) [3 pts.] What is the numerical value of the DC collector current \( I_C \) for this amplifier?
(c) [4 pts.] Find the numerical value of the input resistance $R_{in}$ of this amplifier in kΩ.

(d) [4 pts.] Find the numerical value of the output resistance $R_{out}$ in kΩ.

(e) [3 pts.] Find the numerical value two-port parameter $A_v$, the open-circuit voltage gain, for this amplifier.
(f) [4 pts.] Find the overall voltage gain \( v_{out}/v_i \) with \( R_S \) and \( R_L \) present (values of which are given next to the schematic on the previous page). If you couldn’t solve (a), (b), or (c), you can assume that \( R_{in} = 7 \, k\Omega \), \( R_{out} = 5 \, k\Omega \), and \( A_v = 0.8 \). Needless to say, these are not correct answers to (a), (b), or (c).

(g) [3 pts.] Suppose that the input voltage \( v_i(t) = \dot{v}_i \cos(\omega t) \). What is the maximum amplitude \( \dot{v}_i \) for which the small-signal, two-port model you’ve derived in parts (b)-(c) is reasonably accurate? You can assume that the frequency of \( v_i(t) \) is low enough that capacitors can be neglected. Justify your answer.
3. npn bipolar transistors [10 pts.]

Given:
- Base width $W_B = 150$ nm = 0.15 μm
- Emitter-base junction area $A_E = 25$ μm²
- Emitter width $W_E = 100$ nm = 0.1 μm
- Base-collector junction area $A_C = 50$ μm²
- Electron diffusion constant in base: $D_n = 20$ cm²/s
- Hole diffusion constant in emitter: $D_p = 7$ cm²/s
- Electron charge: $q = 1.6 \times 10^{-19}$ C
- Intrinsic concentration: $n_i = 10^{10}$ cm⁻³

(a) [4 pts.] The collector current for this forward-active npn bipolar transistor is $I_C = 25$ μA. From the cross section of the device shown above, find the numerical value of the minority electron concentration at $x = 0$, at the base side of the emitter-base depletion region.
(b) [3 pts.] For the bias conditions in part (a), the base-emitter voltage $V_{BE} = 697.5$ mV. What is the doping concentration $N_d$ in the base? If you couldn't solve part (a), you can use $n_{pB}(0) = 8 \times 10^{14}$ cm$^{-3}$, which is not the correct answer to part (a), of course.

(c) [3 pts.] The minority hole concentration in the emitter at the edge of the emitter-base depletion region is $(0.04)\ast$ (your answer to part (a)). What is the forward-active DC current gain $\beta_f$ for this transistor? Note that you don't need to have answered part (a) in order to answer this part!