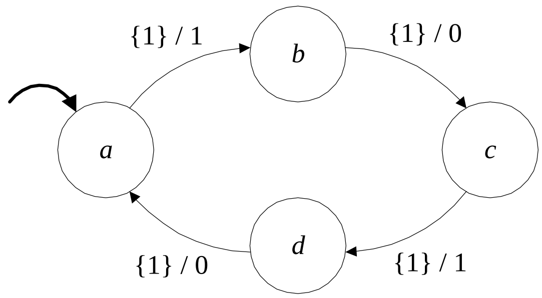
EECS 20. Midterm No. 1 Solution, February 23, 2000.

1. **40 points.** Consider the state machine below



where

$$Inputs = \{1, absent\} \quad \text{and} \quad Outputs = \{0, 1, absent\}$$

(a) Is this machine deterministic or nondeterministic?

Answer:

Deterministic.

(b) Give the update table.

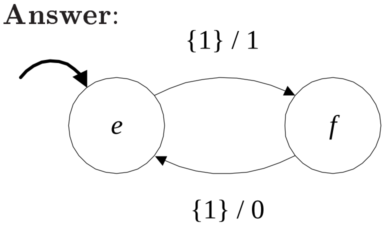
Answer:

The *update* function is given by:

state	(next state, output)	
	1	*absent*
a	(b,1)	(a,*absent*)
b	(c,0)	(b,*absent*)
c	(d,1)	(c,*absent*)
d	(a,0)	(d,*absent*)

(c) Find a deterministic state machine that is bisimilar to this one and has only two states Give it as a state transition diagram by completing the diagram below:

Answer:



(d) Give the bisimulation relation.

Answer:

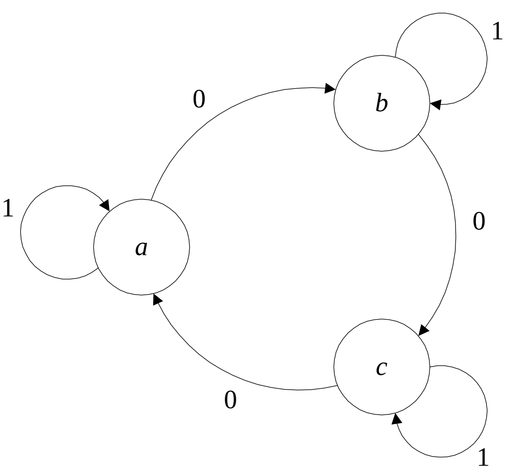
The bisimulation relation is

$$S = \{(a, e), (b, f), (c, e), (d, f)\},$$

or equivalently,

$$S' = \{(e, a), (e, b), (f, c), (f, d)\},$$

2. **30 points.** Let $X = \{a, b, c\}$ represent a set of circles in the following picture:



Consider the following relations, all subsets of $X \times X$:

$$\begin{aligned}
F_0 &= \{(x_1, x_2) \mid \text{there is an arc going from } x_1 \text{ to } x_2 \text{ with a } 0\} \\
F_1 &= \{(x_1, x_2) \mid \text{there is an arc going from } x_1 \text{ to } x_2 \text{ with a } 1\} \\
F_{0and1} &= \{(x_1, x_2) \mid \text{there are two arcs going from } x_1 \text{ to } x_2\text{, one with a } 0 \text{ and one with a } 1\} \\
F_{0or1} &= \{(x_1, x_2) \mid \text{there is an arc going from } x_1 \text{ to } x_2 \text{ with a } 0 \text{ or one with a } 1\}
\end{aligned}$$

(a) Give the elements of the four relations.

Answer:

$$\begin{aligned}
F_0 &= \{(a, b), (b, c), (c, a)\} \\
F_1 &= \{(a, a), (b, b), (c, c)\} \\
F_{0and1} &= \emptyset \\
F_{0or1} &= \{(a, b), (b, c), (c, a), (a, a), (b, b), (c, c)\}
\end{aligned}$$

(b) Which of the four relations are the graph of a function of the form $f \colon X \to X$? List **all** that are such a graph.

Answer: F_0 and F_1.

(c) Are the following assertions true or false?

$$F_{0and1} = F_0 \cap F_1$$

$$F_{0or1} = F_0 \cup F_1$$

Answer:
Both are true.

3. **20 points** Consider all state machines with

$$Inputs = \{1, 2, absent\} \quad \text{and} \quad Outputs = \{1, 2, absent\}$$

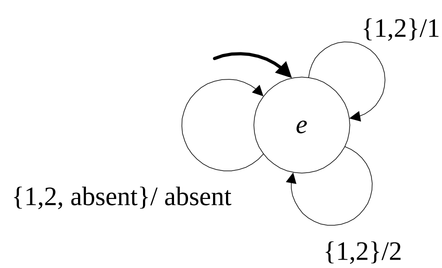
$$States = \{a, b, c, d\}.$$

Assume all these state machines stutter, as usual, when presented with the stuttering input, *absent*.

(a) Give a state machine B that simulates all of these state machines. You will lose points if your machine is more complicated than it needs to be.

(b) Give the simulation relation.

Answer



The simulation relation is

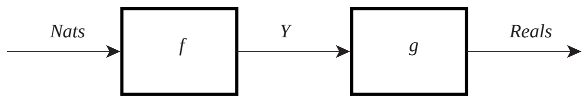
$$S = \{(a, e), (b, e), (c, e), (d, e)\}.$$

4. **30 points** Consider the functions

$$g: Y \to Reals \quad \text{and} \quad f: Nats \to Y.$$

where Y is a set.

(a) Draw a block diagram for $(g \circ f)$, with one block for each of g and f, and label the inputs and output of the blocks with the domain and range of g and f.



(b) Suppose Y is given by

$$Y = [\{1, \cdots, 100\} \to \mathit{Reals}]$$

(Thus, the function f takes a natural number and returns a sequence of length 100, while the function g takes a sequence of length 100 and returns a real number.)

Suppose further that g is given by: for all $y \in Y$,

$$g(y) = \sum_{i=1}^{100} y(i) = y(1) + y(2) + \cdots + y(100),$$

and f by: for all $x \in \mathit{Nats}$ and $z \in \{1, \cdots, 100\}$,

$$(f(x))(z) = \cos(2\pi z/x).$$

(Thus, x gives the period of a cosine waveform, and f gives 100 samples of that waveform.) Give a one-line Matlab expression that evaluates $(g \circ f)(x)$ for any $x \in \mathit{Nats}$. Assume the value of x is already in a Matlab variable called `x`.

Answer:

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sum(cos(2*pi*[1:100]/x))
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(c) Find $(g \circ f)(1)$.

Answer: 100