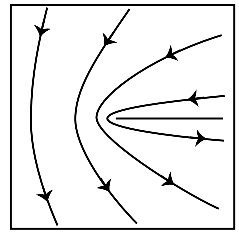
Ö. Savaş NAME: **SOLUTIONS**

11:10-12:30, Thursday, April 8, 2004

ME 106.2
FLUID MECHANICS
EXAM 2 – open book

1.(25%) Consider the flow field whose stream function is given by $\psi = 2\sqrt{r}\ sin(\theta/2)$. Sketch the streamlines passing through $(r,\theta) = (1,0)$ & $(1,\pi)$. Determine the vorticity of the flow field. Construct the corresponding potential function, if possible. If not, explain. (5+5+5+10)

Hint: Vorticty in cylindrical coordinates $\omega = (\partial(ru_\theta)/\partial r)/r - (\partial u_r/\partial\theta)/r$

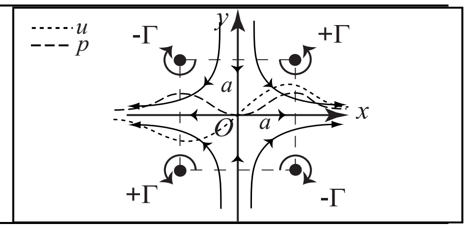


$\psi = 2\sqrt{r}\ sin(\theta/2)$ $(u_r, u_\theta) = (\frac{1}{r}\frac{\partial\psi}{\partial\theta}, -\frac{\partial\psi}{\partial r}) = (\frac{1}{\sqrt{r}}cos(\theta/2), -\frac{1}{\sqrt{r}}sin(\theta/2))$, $\omega = 0$, irrotational flow, potential function exists, $(u_r, u_\theta) = (\frac{\partial\phi}{\partial r}, \frac{1}{r}\frac{\partial\phi}{\partial\theta}) \Longrightarrow \phi = 2\sqrt{r}\ cos(\theta/2)$,

2.(25%) Capillary waves form due to interaction of surface tension and inertial forces. Let us consider the speed of these waves on water surface. For very short wave lengths λ, the wave speed C is independent of water depth. Obtain the form of the dependence of C on the surface tension σ, λ and relevant fluid properties. (25)

$$C \sim (3\pi)(\sigma/\rho)^{1/2}\lambda^{-1/2}$$

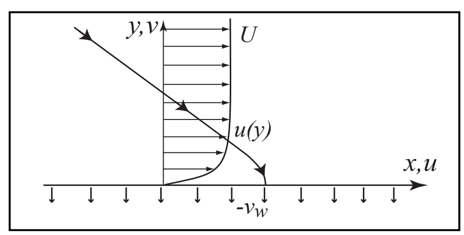
3.(25%) Consider the four symmetrically placed equi-strength vortices shown in the figure. Note the alternating senses of the vortices. Sketch the stream line pattern of the flow field. In particular, draw the streamlines passing through the origin. Determine the velocity long the x-axis. Determine the pressure distribution along the x-axis. (5+10+10) *Hint: Exploit the symmetry in the problem.*



$$u = \frac{\Gamma}{\pi a}\left[\frac{1}{(\xi-1)^2+1} - \frac{1}{(\xi+1)^2+1}\right] = \frac{4\Gamma}{\pi a}\frac{\xi}{[(\xi-1)^2+1][(\xi+1)^2+1]} \quad \text{where} \quad \xi = x/a$$

$$C_p \equiv= \frac{p-p_\infty}{\rho\Gamma^2/a^2} = \frac{16}{\pi^2}\frac{\xi^2}{[(\xi-1)^2+1]^2[(\xi+1)^2+1]^2}$$

4.(25%) Consider the incompressible, viscous, steady, 2D flow over an infinitely long flat plate. The velocity component parallel to the wall is U far away from the wall. There is uniform suction at the wall, that is, flow is drawn into the wall at $-v_w$ everywhere. Since the flow is of infinite extend, there is no x dependence. Using the continuity equation, show that the vertical velocity component v is uniform. Starting from the x-momentum equation, obtain the ordinary differential equation for $u(y)$. State the boundary conditions. Integrate the equation to determine the horizontal velocity profile $u(y)$. Sketch the streamlines. (5+5+5+5+5)



Asymptotic suction:

$$\partial/\partial x \equiv 0 \Longrightarrow \partial v/\partial y = 0 \Longrightarrow v = -v_w \Longrightarrow -v_w\frac{du}{dy} = \nu\frac{d^2u}{dy^2} \Longrightarrow \frac{d}{dy}\left(e^{v_w y/\nu}\frac{du}{dy}\right) = 0$$

$$u(0) = 0, u(\infty) = U \Longrightarrow \frac{u(y)}{U} = 1 - e^{-v_w y/\nu}$$