Physics 7A First Exam July 24, 2000

No books or papers may be used except one (1) card, not larger than 3"x5" (both sides). The exam totals 100 points.

(10) (1) A car is speeding toward a town at 150 km per hour. The car is stopped by the highway patrol. The driver is told that, if she drove at the (legal) speed of 90 km per hour, she would arrive at the town 3 minutes later than she would if she drove at 150 km per hour. Calculate the distance from the town at which the car was stopped.

(10) (2) A mass m starts from rest at the top of an incline whose vertical height is h, and slides to the bottom. The incline surface is frictionless and makes an angle θ with the horizontal. (a) Calculate the speed v of the mass at the bottom of the incline; (b) Show explicitly that your answer in (a) has the correct units [(a) = 9 points, (b) = 1 point].

(15) (3) A cannon is located at the top of a vertical cliff of height h; the barrel of the cannon makes an angle θ with the horizontal. The cannon fires a projectile with an initial velocity of magnitude v_0 and the projectile lands a horizontal distance R from the foot of the cliff. Calculate h.

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(15)(4) A particle of mass 1 kg is moving with angular speed 1 radian $(\text{sec})^{-1}$ in a circle of radius 2 meters. At time $t=0$, a tangential force of constant magnitude 10 Newtons is applied to the particle, and continues to be applied.
(a) Calculate the total acceleration vector $\underline{a}$ at a time 1 second after the force is first applied. (Your answer should be in terms of unit vectors in plane polar coordinates.)
(b) Calculate the magnitude of $\underline{a}$ at $t=1$ sec. [(a)=12, (b)=3].

(20)(5) A bullet of mass m is fired horizontally with speed v into a wood block of mass M which is initially at rest on a surface. The bullet becomes embedded in the block and the block moves a distance d from its initial position. The coefficient of friction between the block and the surface on which it moves is μ. Calculate the speed v of the bullet.

(30)(6) A uniform piece of wire of mass M is bent into a semicircle of radius R. Calculate explicitly the coordinates (x_{cm}, y_{cm}) of the center of mass of the wire with respect to an origin O which is the center of the diameter of the semicircle.

Physics 7A   Solutions to Exam #1   7-24-00

(1) Let d = distance from the town
t_f = time for car to travel distance d at 150 km/hr
t_s = time for car to travel distance d at 90 km/hr

Then, since distance equals speed times time,

$$d = 150\, t_f \qquad (t_f \text{ in hours})$$
$$d = 90\, t_s \qquad (t_s \text{ in hours})$$

We also know that t_s is 3 minutes $(= \frac{1}{20} = 0.05 \text{ hour})$ longer than t_f, so

$$t_s = t_f + 0.05$$

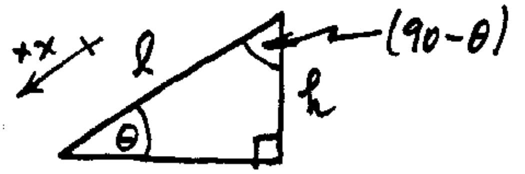
and equations above become

$$150\, t_f = 90\,(t_f + 0.05)$$
$$t_f = 0.075 \text{ hour} = 4.5 \text{ minutes}$$

Then the required distance d is given by

$$d = 150\, t_f = (150)(0.075) \text{ km}$$
$$\boxed{d = 11.25 \text{ km}}$$

(2) There are several equivalent correct ways to do this:



$h = l \sin\theta$

$\boxed{\begin{array}{l} \text{(b)} \\ g \text{ in m sec}^{-2} \\ h \text{ in m} \\ (gh)^{1/2} \text{ in m sec}^{-1} \end{array}}$

Method 1: Acceleration due to gravity down the incline has constant value $(g\sin\theta)$, so

$$v^2 = v_0^2 + 2ax \quad \text{with} \begin{cases} v_0 = 0 \text{ (rest)} \\ a = g\sin\theta \\ x = l \end{cases}$$

$$v^2 = 2(g\sin\theta)l$$

$$v^2 = 2g(l\sin\theta)$$

$$v^2 = 2gh \implies \boxed{v = (2gh)^{1/2}}$$

Method 2: Since acceleration is constant, we have

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \implies l = 0 + 0 + \tfrac{1}{2}(g\sin\theta)t^2$$

where $v_0 = 0$ (rest) and we set $x_0 = 0$ at top of incline. Then, at bottom, block reaches bottom at time given by

$t = \left(\frac{2l}{g\sin\theta}\right)^{1/2}$. Speed v at time t is $v(t) = v_0 + at$, or

$$v(\text{bottom}) = at = (g\sin\theta)\left(\frac{2l}{g\sin\theta}\right)^{1/2}$$

$$v^2(\text{bottom}) = (g^2\sin^2\theta)\left(\frac{2l}{g\sin\theta}\right) = 2gl\sin\theta$$

Since $h = l\sin\theta$, $\boxed{v(\text{bottom}) = (2gh)^{1/2}}$

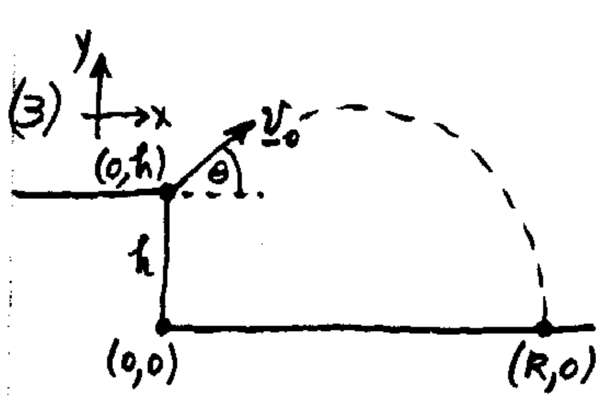
Method 3: Total energy E is conserved; U = gravitational potential energy, so $T(\text{top}) + U(\text{top}) = T(\text{bott.}) + U(\text{bott.})$

$$0 + mgh = \tfrac{1}{2}mv^2 + 0$$

where we choose $U \equiv 0$ at bottom of the incline. Then solve for v: $\boxed{v = (2gh)^{1/2}}$

All three methods are equivalent and correct.

(3)



$v_{0x} = v_0 \cos\theta$
$v_{0y} = v_0 \sin\theta$

For projectile: $\begin{cases} x(t) = x_0 + v_{0x} t \\ y(t) = y_0 + v_{0y} t - \frac{1}{2} g t^2 \end{cases}$

Since (see diagram) we have set $x_0 = 0$, $y_0 = h$, then at the time t at which projectile hits the ground,

$$\begin{cases} R = (v_0 \cos\theta) t & (1) \\ 0 = h + (v_0 \sin\theta) t - \frac{1}{2} g t^2 & (2) \end{cases}$$

so time at which projectile hits is $t = \frac{R}{(v_0 \cos\theta)}$ from (1)
Then from (2),

$$0 = h + (v_0 \sin\theta)\left(\frac{R}{v_0 \cos\theta}\right) - \frac{1}{2} g \frac{R^2}{v_0^2 \cos^2\theta}$$

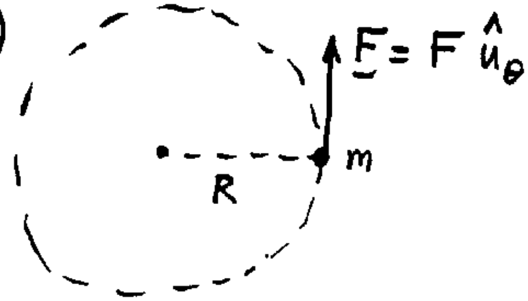
$$0 = h + R\tan\theta - \frac{gR^2}{2v_0^2 \cos^2\theta}$$

$$\boxed{h = \frac{gR^2}{2v_0^2 \cos^2\theta} - R\tan\theta}$$

Dimensional check (not required): R in meters,

$\frac{gR^2}{v_0^2 \cos^2\theta}$ in $\frac{m\, sec^{-2}\, m^2}{m^2\, sec^{-2}}$ = meters so h is in meters

(4)



Since $F = |\underline{F}|$ is constant, torque $\tau = RF = I\alpha$ is const. so angular acceleration α is constant. Hence angular velocity $\omega(t) = \omega_0 + \alpha t$

Tangential velocity $v_T(t) = R\omega(t)$

Tangential acceleration $a_T = R\alpha = \text{const.}$

Radial acceleration $a_R(t) = R[\omega(t)]^2$

(a) From Newton's 2nd Law: $\underline{a}_T = (F/m)\hat{u}_\theta$ since $\underline{F} = F\hat{u}_\theta$

and $\underline{a}_R = (R\omega^2)(-\hat{u}_r)$

Since $a_T = |\underline{a}_T| = (F/m) = R\alpha \Rightarrow \boxed{\alpha = (F/mR)}$

Then $\omega(t) = \omega_0 + \alpha t = \omega_0 + (F/mR)t$

$$\omega^2 = [\omega_0 + (F/mR)t]^2$$

$$\underline{a}_R = R\omega^2(-\hat{u}_r) = R[\omega_0 + (F/mR)t]^2(-\hat{u}_r)$$

Total acceleration $\boxed{\underline{a} = \underline{a}_R + \underline{a}_T = \left\{\left(\frac{F}{m}\right)\hat{u}_\theta + R\left[\omega_0 + \left(\frac{F}{mR}\right)t\right]^2(-\hat{u}_r)\right\}}$

If $m = 1\text{ kg}$, $\omega_0 = 1\text{ sec}^{-1}$, $F = 10\text{ N.}$, $R = 2\text{ m.}$, so, when $t = 1\text{ sec}$,

$$\boxed{\underline{a} = [10\,\hat{u}_\theta + 72(-\hat{u}_r)]}\ \text{m sec}^{-2}$$

(b) $|\underline{a}|^2 = (\underline{a}\cdot\underline{a}) = [100 + 5184]$ since $\begin{cases}\hat{u}_\theta\cdot\hat{u}_r = 0\\ \hat{u}_\theta\cdot\hat{u}_\theta = 1,\ \text{etc.}\end{cases}$

$$|\underline{a}| = (5284)^{1/2}$$

$\boxed{|\underline{a}| = 73\ \text{m sec}^{-2}}$ when $t = 1$ sec.

(5) After impact, the block plus bullet form a mass $(M+m)$ moving with velocity v', starting from rest. Since no external forces act on the block or bullet before and just after the collision, linear momentum is conserved during the collision. (The frictional force doesn't act on the block until just after the block plus bullet start to move.) From conservation of linear momentum,

$$mv = (M+m)v' \qquad (1)$$

where v' is the initial (just after collision) velocity of (block + bullet). Using the relation $v^2 = v_0^2 + 2ax$, we get

$$0 = v'^2 + 2ad \qquad (2)$$

because the (block + bullet) stop $(v=0)$ after they have moved a distance d. From (1) and (2),

$$0 = \left(\frac{mv}{(M+m)}\right)^2 + 2ad$$

To solve for v, we need the acceleration a of the (block + bullet). The only force acting on (block + bullet) during its (post-collision) motion is the frictional force

$$f = \mu N = \mu(M+m)g$$

so the acceleration (directed opposite to the motion) is, from $f = -\mu(M+m)g = (M+m)a$,

$$a = -\mu g$$

Then the velocity v of the bullet (before collision) is given by

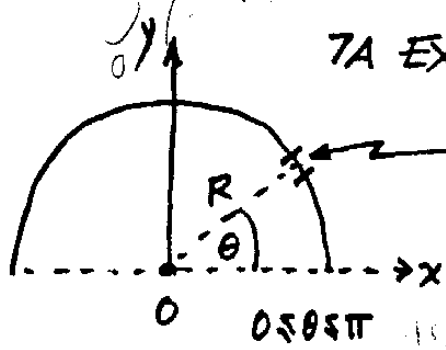
$$0 = \frac{m^2 v^2}{(M+m)^2} - 2\mu g d$$

so

$$\boxed{v = \left(\frac{M+m}{m}\right)(2\mu g d)^{1/2}}$$

$\int_0^{\pi} \rho\, R d\theta \cdot R \sin\theta$

(6)



$dm = \lambda\, ds$ where λ is the linear mass density of the wire in $kg\, m^{-1}$

$\boxed{ds = R d\theta}$

$0 \leq \theta \leq \pi$ and $\lambda = (M/\pi R) \Rightarrow \boxed{M = \pi R \lambda}$

$\boxed{dm = \lambda R\, d\theta}$

Coordinates of cm:

$$y_{cm} = \frac{1}{M}\int y\, dm = \frac{1}{\pi R \lambda}\int R \sin\theta\, (\lambda R\, d\theta) = \frac{\lambda R^2}{\pi R \lambda}\int_0^{\pi} \sin\theta\, d\theta$$

$$y_{cm} = \frac{R}{\pi}\Big[-\cos\theta\Big]_0^{\pi} = \frac{R}{\pi}\Big[-(\cos\pi - \cos 0)\Big] = \frac{R}{\pi}\Big[-[-1 \quad -1]\Big]$$

$$\boxed{y_{cm} = (2R/\pi) = 0.637R}$$

$$x_{cm} = \frac{1}{M}\int x\, dm = \frac{1}{\pi R \lambda}\int_0^{\pi} R\cos\theta\, (\lambda R) d\theta = \frac{\lambda R^2}{\lambda \pi R}\int_0^{\pi} \cos\theta\, d\theta$$

$$x_{cm} = \frac{R}{\pi}\int_0^{\pi} \cos\theta\, d\theta = \frac{R}{\pi}\Big[\sin\theta\Big]_0^{\pi} = \frac{R}{\pi}\Big[\sin\pi - \sin 0\Big] = \frac{R}{\pi}(0-0)$$

$$\boxed{x_{cm} = 0}$$

The center of mass lies on the y-axis (since $x_{cm} = 0$) at a distance $y_{cm} = (2R/\pi) = 0.637R$ from point O.