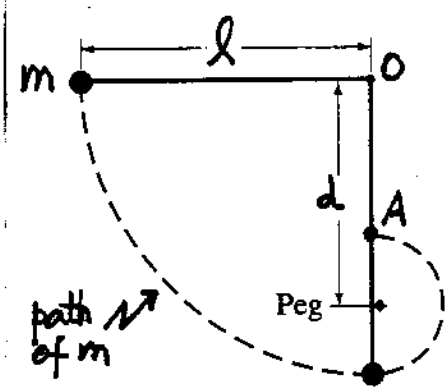
Physics 7A (Sec 2) Midterm Exam #2 Nov. 5, 2002

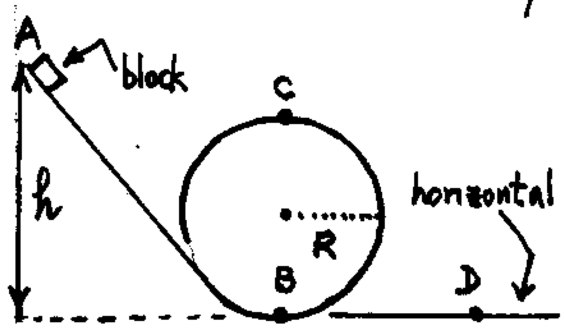
You may use two (2) cards, 3"x 5", as memory aids. Exam = 200 points

(30)(1) A small particle of mass m is held horizontal at the end of a massless string of length $l$, as shown. A peg is located at




a distance d vertically below point O where the string is attached. Mass m is released, the string catches on the peg (when the string is vertical and the mass m describes a new circular path with the peg as its center. In order to complete this new circular path, the tension in the string must be non-zero when the mass reaches point A. There exists a critical value $d_c$ of the distance d for which the tension in the string is zero at point A. Calculate the value of $d_c$ in terms of $l$.

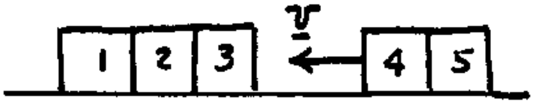
(30)(2) Given a "loop-the-loop" as shown, with a circular loop of radius R. A small block of mass m slides (without friction, starting




from rest at point A. (a) Calculate the minimum value of h for which the block will remain on the track; (b) If $N_C$ is the magnitude of the normal force exerted by the track on the block at point C, and $N_B$ is the magnitude of the normal force exerted on the block at point B, calculate $(N_B - N_C)$ in terms of m and g; (c) Calculate the speed $v_D$ of the block at point D; (d) Does $(N_B - N_C)$ depend on h and/or R? Justify your answer. [Part (a) = 10, (b) = 10, (c) = 5, (d) = 5 points]

(continued ⟶)

(30)(3) Three identical blocks (numbered 1, 2, 3), each of mass $m$, are at rest and in contact on a frictionless horizontal surface, as shown. Two identical blocks (4, 5) are also identical to blocks 1, 2, 3 and are in contact.

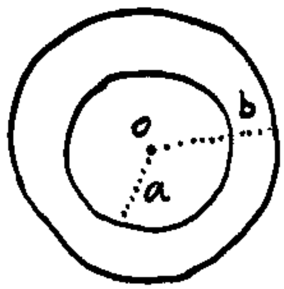



Blocks 4 and 5 move with speed $v$, collide elastically with block 3 (which remains at rest throughout); blocks 4 and 5 come to rest after the collision. Prove that, after the collision, blocks 1 and 2 (still in contact) move off to the left with speed $v'$ instead of block 1 moving off with speed $(2v')$.

(30)(4) Given a symmetric three-dimensional rigid body (for example, a steel sphere) at rest at the top of an incline so that the center of mass of the body is at a height $h$ above the bottom of the incline. The body rolls down the incline without slipping. (a) Show that, at the bottom of the incline, the speed $v_{cm}$ of the center of mass of the body is independent of the mass and dimensions of the body; (b) Describe a possible experiment to measure (using your answer to (a)) the numerical pre-factor in the expression for the moment of inertia (relative to the center of mass) of a rigid body of arbitrary geometric shape. [(a)=25, (b)=5 pts]

(continued →)

(40) (5) Given a thin solid circular annular ring of inner radius a and outer radius b ($b > a$), as shown. The area density of the material of the ring is $\sigma$ kg m$^{-2}$. Consider a point mass m at a vertical distance z above the plane of the ring and on the axis (passing through point O) of the ring. Calculate the magnitude and direction of $\underline{F}(z)$, the gravitational force exerted on mass m by the ring.




(40) (6) Given a thin solid plane quarter-circular disc of radius R and of area density $\sigma$ kg m$^{-2}$. Calculate the moment of inertia $I_{cm}$ about an axis which passes through the center of mass of the disc and perpendicular to the plane of the disc. Express your answer in terms of R and the mass M of the disc.

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## MATHEMATICAL FACTS

Elements of area: $dA = r\,dr\,d\theta$ ; $dA = dx\,dy$ ; $dA = 2\pi r\,dr$

Integrals: $$\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = \frac{-1}{(x^2+a^2)^{1/2}} + C$$

$$\int \frac{x\,dx}{(x^2+a^2)^{1/2}} = (x^2+a^2)^{1/2} + C$$

(a = const.)

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{(x^2+a^2)^{1/2}} + C$$

$$\int \frac{dx}{(x^2+a^2)^{1/2}} = \ln\left(x+(x^2+a^2)^{1/2}\right) + C$$